Heep Yunn School

Final Examination (2019-2020)

Form 3 Mathematics

Question Paper

F. 3A, B, C, D, E July 2020

Time allowed: 1 hour

Total: 50 marks

Name: ______________________ Class: 3____ Class Number: ______

Instructions

1. Do not turn over the exam paper until you are told to do so.
2. Attempt ALL questions in this paper. All the answers should be written on the respective answer sheets.
3. Write down your Name, Class and Class number on the first page of the Question Paper and the Question-Answer Booklet.
4. Choose one answer to each question in Section A. Two or more answers will score NO MARKS.
5. All questions in Section A carry equal marks. No marks will be deducted for wrong answers.
6. Only the working written on the Question-Answer Booklets will be marked.
7. In Sections B and C, all working must be clearly shown. Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.
8. The diagrams in this paper are not necessary drawn to scale.

Section A: Multiple Choice Questions (10 marks)

1. $36^{500} \times (-6)^{-1200} =$

   A. 6^{200}

   B. -6^{200}

   C. $\frac{1}{6^{200}}$

   D. $-\frac{1}{6^{200}}$

2. If $-5-4x \geq \frac{2x-1}{3}$, then

   A. $x \leq -1$.

   B. $x \leq 1$.

   C. $x \geq -1$.

   D. $x \geq 1$.

3. A sum of $25000 is deposited into a bank at an interest rate of 4% per annum, compounded every 4 months. Find the amount after 4 years.

   A. $28171, correct to the nearest dollar

   B. $29307, correct to the nearest dollar

   C. $29314, correct to the nearest dollar

   D. $30901, correct to the nearest dollar

4. Three fair coins are thrown. Find the probability that at least two heads show up.

   A. $\frac{1}{2}$

   B. $\frac{1}{3}$

   C. $\frac{1}{4}$

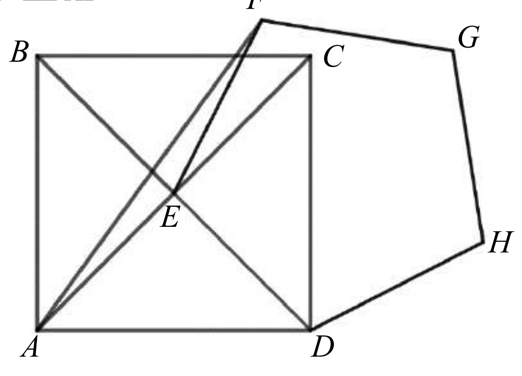
   D. $\frac{3}{8}$

5. Office O is 6 km due south of Cinema C and 12 km due east of Market M. Find the bearing of M from C.
   A. N27°E (correct to the nearest degree)
   B. N63°E (correct to the nearest degree)
   C. S27°W (correct to the nearest degree)
   D. S63°W (correct to the nearest degree)

6. The distance between the points $(a, 0)$ and $(0, -3a)$, where a is positive, is
   A. $2a$.
   B. $\sqrt{8}a$.
   C. $\sqrt{10}a$.
   D. $4a$.

7. Which of the lines below are perpendicular to the line joining $A(5,5)$ and $B(2,11)$?
   I. A line parallel to the line through the origin and $C(-6,-3)$
   II. A line with inclination 30°
   III. The line passing through A and $D(21,13)$
   A. I and II only
   B. I and III only
   C. II and III only
   D. I, II and III

8. In the figure, $ABCD$ is a rectangle with diagonals intersecting at E. $DEFGH$ is a regular pentagon. If $\angle CEF = 38°$, then $\angle BAF =$
   A. 16°.
   B. 19°.
   C. 26°.
   D. 36°.

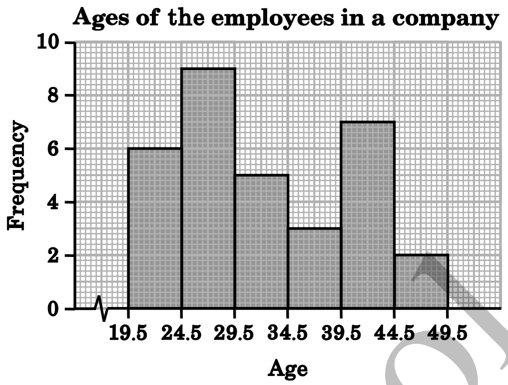


9. The histogram shows the distribution of the ages of the employees in a company. Which of the following descriptions must be correct?

   I. The age of the youngest employee is 20.

   II. The difference in the ages of the oldest and second oldest employees is at most 5.

   III. There are fewer employees of ages less than 29.5 than there are those of ages not less than 29.5.



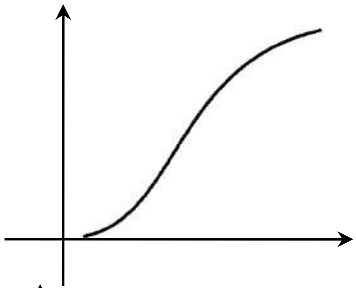
A. I only

B. II only

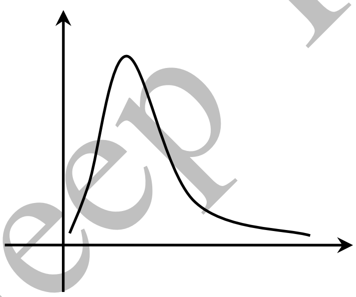
C. I and II only

D. II and III only

10. The cumulative frequency curve on the right shows the distribution of a set of data. Which of the following curves can be the frequency curve of the same set of data?



A.



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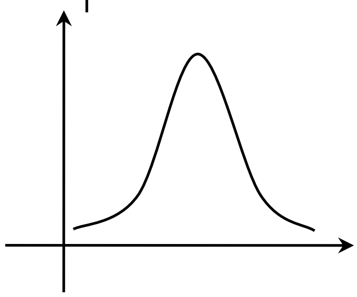

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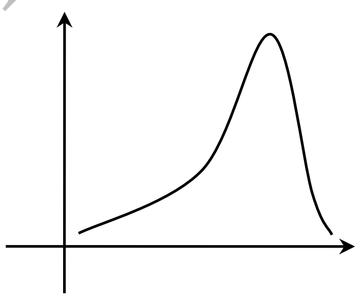

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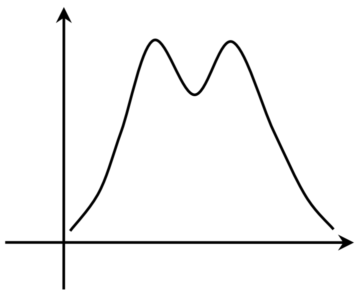


End of Section A

Refer to the Question-Answer Booklet for Sections B and C.

Heep Yunn School

Final Examination (2019-2020)

Form 3 Mathematics

Question-Answer Booklet Mark: ______/50

Name: ______________________________ Class: 3____ Class Number: ______

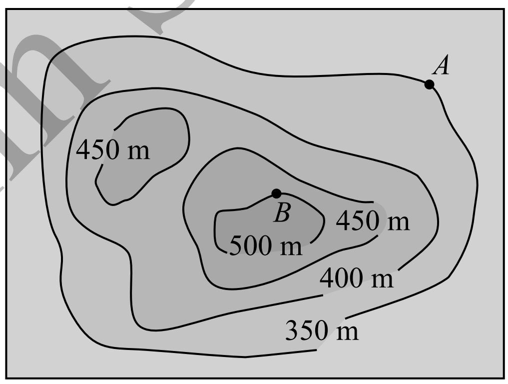
Section A: Multiple Choice Questions (10 marks)

Write the letter (A, B, C or D) in the corresponding box for each question below.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.

Section B: Short Questions (15 marks)

1. The figure shows a contour map of scale $1:32000$. A and B are two pavilions. Their distance on the map is 6 cm. If a straight path is constructed connecting A and B, find the gradient of the path. (Express your answer in the form $1:n$, where n is correct to the nearest integer.)

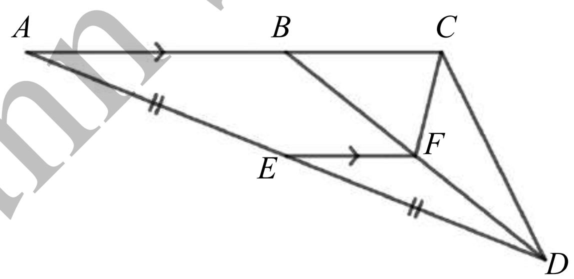


(2 marks)

2. Find the inclination of the line joining $A(-10, 6)$ and $B(3, 13)$ correct to the nearest degree.

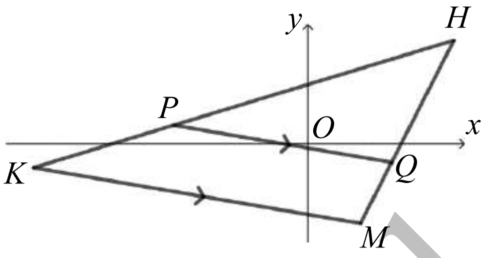
(2 marks)

3. In the figure, *ABC*, *AED* and *BFD* are straight lines. *E* is the mid-point of *AD*. *AC* and *EF* are parallel. What special line of ΔBCD is line *CF*? Explain your answer.



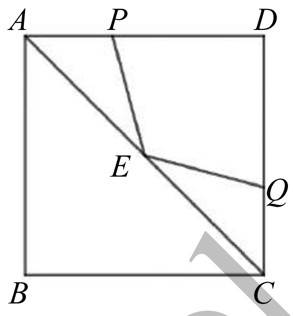
(3 marks)

4. In the rectangular coordinate system shown in the figure, $\Delta HKM \sim \Delta HPQ$ and $PQ // KM$. The coordinates of H, P and K are $(25, 39)$, $(-23, 7)$ and $(a, -9)$ respectively. Find the ratio $\dfrac{\text{Area of } \Delta HPQ}{\text{Area of } \Delta HKM}$.



(4 marks)

5. In the figure, $ABCD$ is a square. E is the point of intersection of the diagonals BD (not shown) and AC. P and Q are points on AD and CD respectively. $\angle CEQ = 30°$ and $\angle EPD = 75°$.
Prove that $EP = EQ$.



(4 marks)

Section C: Conventional Questions (25 marks)

1. The frequency distribution table and cumulative frequency table below show the distribution of the heights (in m) of some trees in a garden.

Height (m)	*Frequency*
7 – 10	a
11 – 14	12
15 – 18	b
19 – 22	4

Height less than (m)	*Cumulative frequency*
10.5	6
14.5	b
18.5	c
22.5	40

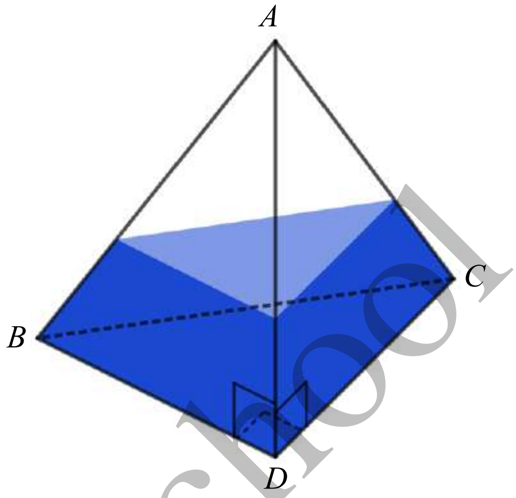
(a) Find the values of a, b and c.

(3 marks)

(b) If a tree is randomly selected, find the probability that the height of the tree is less than 18.5 m but not less than 10.5 m.

(2 marks)

2. The figure shows a sealed vessel $ABCD$ containing 1520 cm^3 of water. The faces ABD, ACD and BCD are right-angled at D. $AD = 30$ cm. When the vessel is placed with face BCD on the horizontal ground, the depth of water is 10 cm.



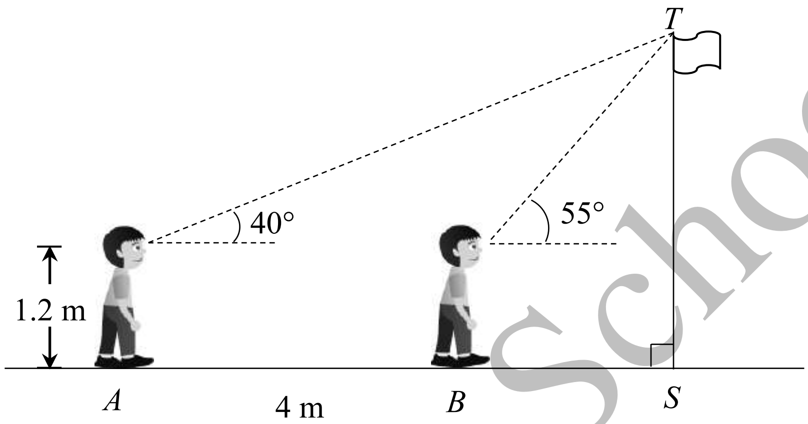
(a) Find the capacity of the whole vessel.

(3 marks)

(b) Suppose the vessel is now placed with face ACD on the horizontal ground. If $BD = x$ cm, express the depth of water in terms of x.

(2 marks)

3. Paul is measuring the height of a flagpole ST, as shown in the figure. His eyes are 1.2 m above the ground. When he looks at the top T of the flagpole from a position A, the angle of elevation is 40°. When he moves 4 m closer to the



flagpole to another position B, the angle of elevation becomes 55°.

(a) Let $ST = h$ m. Express AS and BS in terms of h.

(3 marks)

(b) Find the height of the flagpole.

(2 marks)

(c) Paul claims that if he walks closer to the flagpole by a further 4 m, the angle of elevation of the top of the flagpole will be increased by another 15°. Do you agree with him? Explain your answer.

(2 marks)

4. (a) If the diagonals of a quadrilateral *ABCD* bisect each other at *E* and are perpendicular to each other, prove that *ABCD* is a rhombus.

(3 marks)

(b) In a rectangular coordinate plane, the coordinates of *O*, *P*, *Q* and *R* are $(0,0)$, (h,k), $(h+k,h+k)$ and (k,h) respectively, where h and k are two non-zero constants.

(i) Using (a), show that *OPQR* is a rhombus.

(ii) Pauline claims that *OPQR* is a square. Do you agree with her? Explain your answer.

(5 marks)

End of Paper

Heep Yunn School
Final Examination (2019-2020)
Form 3 Mathematics
SOLUTIONS

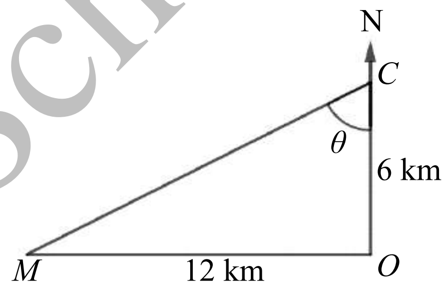
Section A: Multiple Choice Questions (10 marks): C A B A D C B D D C

1. $36^{500} \times (-6)^{-1200} = (6^2)^{500} \times 6^{-1200} = 6^{2(500)+(-1200)} = 6^{-200} = \dfrac{1}{6^{200}}$

2. $-5-4x \ge \dfrac{2x-1}{3} \quad \Rightarrow \quad -15-12x \ge 2x-1 \quad \Rightarrow \quad -14x \ge 14 \quad \Rightarrow \quad x \le -1$

3. $\text{Amount} = \$25000\left(1+\dfrac{4\%}{3}\right)^{4\times 3} = \$29307$ (nearest dollar)

4. All outcomes: HHH HHT HTH THH TTH THT HTT TTT
   Favourable outcomes: HHH HHT HTH THH
   Hence, required probability $= \dfrac{4}{8} = \dfrac{1}{2}$

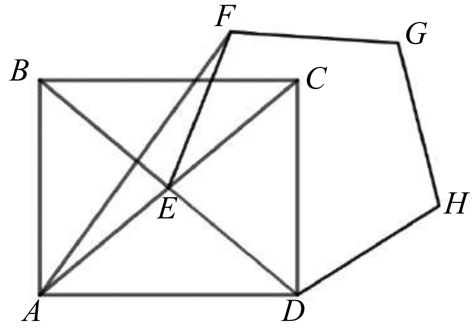


5. $\tan\theta = \dfrac{12}{6} \quad \Rightarrow \quad \theta = 63^\circ$ (nearest degree)
   $\therefore$ Bearing $=$ S63°W

6. $\text{Distance} = \sqrt{(a-0)^2+(0+3a)^2} = \sqrt{10a^2} = \sqrt{10}a$

7. $m_{AB} = \dfrac{11-5}{2-5} = -2 \quad \Rightarrow \quad$ slope of required line $= -1 \div (-2) = \dfrac{1}{2}$
   I. $m_{OC} = \dfrac{0+3}{0+6} = \dfrac{1}{2}$
   II. Slope of line $= \tan 30^\circ = 0.577 \ne 0.5$
   III. $m_{AD} = \dfrac{13-5}{21-5} = \dfrac{1}{2}$

8. $\angle DEF = (5-2)180^\circ \div 5 = 108^\circ$ ($\angle$ sum of polygon)
   $DE = FE$ (given)
   $DE = CE = AE$ (property of rectangle)
   Hence, ΔADE and ΔAEF are both isosceles.
   $\therefore$ In ΔADE, $\angle EAD = \angle ADE$ (base $\angle$s, isos. Δ)
   $= \angle DEC \div 2$ (ext. $\angle$ of Δ)
   $= (108^\circ - 38^\circ) \div 2 = 35^\circ$
   And in ΔAEF, $\angle FAE = \angle AFE$ (base $\angle$s, isos. Δ)
   $= 38^\circ \div 2 = 19^\circ$ (ext. $\angle$ of Δ)
   Therefore, $\angle BAF = \angle BAD - 19^\circ - 35^\circ$
   $= 90^\circ - 19^\circ - 35^\circ = 36^\circ$ (property of rectangle)



9. I. False. $19.5 \le \text{Youngest age} < 24.5$
II. True as both are between 44.5 and 49.5.
III. True. Ages less than 29.5: 15 employees; Ages not less than 29.5: 17

10. A gentle section in the cumulative frequency curve means there are few data in that interval. Thus for this set of data, there are few small and large data, and more data in the middle ones. And the distribution is roughly symmetric.

Section B: Short Questions (15 marks)

1. $\text{Gradient} = \dfrac{500-350}{6\times 32000 \div 100}$ — 1M for $\dfrac{\text{rise}}{\text{run}}$

$= 0.078125 = \dfrac{1}{13}$ (nearest integer) — 1A

(2)

2. $m_{AB} = \dfrac{13-6}{3+10} = \dfrac{7}{13}$ — 1M for $\dfrac{\Delta y}{\Delta x} = \tan\theta$

$\therefore$ Inclination $= \tan^{-1}\dfrac{7}{13}$

$= 28°$ (nearest degree) — 1A

(2)

3. $\because$ $AB // EF$ and $AE = ED$ (given)
$\therefore$ $BF = FD$ (intercept theorem) — 2/1/0
Hence, CF is a median of ΔBCD. — 1

(3)

4. <u>Method 1</u>

Let $HP : PK = r : 1$. — 1M

$\dfrac{(-9)(r)+(39)(1)}{r+1} = 7 \Rightarrow 39-9r = 7r+7 \Rightarrow r = 2$ — 1A

Hence, Required ratio $= \left(\dfrac{HP}{HK}\right)^2 = \left(\dfrac{2}{3}\right)^2 = \dfrac{4}{9}$ — 1M + 1A

<u>Method 2</u>

$m_{KP} = m_{HP}$ — 1M

$\dfrac{7-(-9)}{-23-a} = \dfrac{39-7}{25-(-23)} = \dfrac{2}{3} \Rightarrow a = -47$ — 1A

Hence, $HP = \sqrt{(25+23)^2+(39-7)^2} = \sqrt{3328}$

$HK = \sqrt{(25+47)^2+(39+9)^2} = \sqrt{7488}$

Required ratio $= \left(\dfrac{HP}{HK}\right)^2 = \left(\dfrac{\sqrt{3328}}{\sqrt{7488}}\right)^2 = \dfrac{4}{9}$ — 1M + 1A

(4)

5. In ΔAEP and ΔCEQ,

$\angle PAE = \angle QCE = 45°$ (property of square)

$AE = CE$ (property of square)

$\angle PEA = \angle EPD - \angle PAE$ (ext. $\angle$ of Δ)

$= 75° - 45°$

$= 30° = \angle QEC$ (given)

$\therefore$ $\Delta AEP \cong \Delta CEQ$ (ASA) 3/2/1/0

Hence, $EP = EQ$ (corr. sides, $\cong \Delta$s) 1

(4)

Section C: Conventional Questions (25 marks)

1.(a) $a = 6$ 1A

$b = 6 + 12 = 18$ 1A

$c = 40 - 4 = 36$ 1A

1.(b) Required probability $= \dfrac{12+18}{40} = \dfrac{3}{4}$ 1M + 1A

(5)

2.(a) Let the capacity of the vessel be V cm^3.

$\dfrac{V-1520}{V} = \left(\dfrac{30-10}{30}\right)^3 = \dfrac{8}{27}$ 1M + 1A

$27(V-1520) = 8V \Rightarrow V = 2160$ 1A

Hence, the capacity is 2160 cm^3.

2.(b) Since the volume of water and the capacity of vessel remain unchanged, the linear ratios are also unchanged. 1M

$\therefore$ Depth of water $= \dfrac{x}{3}$ (cm) 1A

(5)

3.(a) $\tan 40° = \dfrac{h-1.2}{AS} \Rightarrow AS = \dfrac{h-1.2}{\tan 40°}$ (m) (or equiv.) 1M for either + 1A

$\tan 55° = \dfrac{h-1.2}{BS} \Rightarrow BS = \dfrac{h-1.2}{\tan 55°}$ (m) (or equiv.) 1A

3.(b) $AS - BS = \dfrac{h-1.2}{\tan 40°} - \dfrac{h-1.2}{\tan 55°}$ 1M for using (a)

$4 = (h-1.2)\left(\dfrac{1}{\tan 40°} - \dfrac{1}{\tan 55°}\right)$

$h = 4 \div \left(\dfrac{1}{\tan 40°} - \dfrac{1}{\tan 55°}\right) + 1.2$

$= 9.34$ (cor. to 3 sig. fig.) 1A

Hence, the height is 9.34 m.

3.(c) The new angle of elevation

$= \tan^{-1}\dfrac{h-1.2}{BS-4} = 78° \neq 55° + 15°$. Hence wrong. 1M + 1

(7)

4.(a) *ABCD* is a parallelogram. (diags. bisect each other) 1

Method 1

Let $AE = CE = x$ and $BE = ED = y$.

$AB = \sqrt{x^2 + y^2}$, and *BC*, *CD* and *DA* are the same.

Hence, *ABCD* is a rhombus. 2/1/0

Method 2

In ΔABE and ΔCBE,

$AE = CE$ (given)

$BE = BE$ (common)

$\angle BEA = \angle BEC = 90°$ (given)

$\Delta ABE \cong \Delta CBE$ (SAS)

Hence, $AB = CB$ (corr. sides, $\cong \Delta$s)

Similarly, $CD = DA = AB = BC$. Hence rhombus. 2/1/0

4.(b) (i) Mid-pt of $PR = \left(\dfrac{h+k}{2}, \dfrac{k+h}{2}\right)$

Mid-pt of $OQ = \left(\dfrac{0+h+k}{2}, \dfrac{0+h+k}{2}\right)$

$= \left(\dfrac{h+k}{2}, \dfrac{h+k}{2}\right) =$ Mid-pt of *PR*

Hence, *PR* and *OQ* bisect each other. ***1 ← (either)***

$m_{PR} = \dfrac{k-h}{h-k} = -1$ 1M

$m_{OQ} = \dfrac{h+k-0}{h+k-0} = 1$

Hence, $m_{PR}m_{OQ} = -1$. Thus perpendicular. ***1 ← (or)***

$\therefore$ *OPQR* is a rhombus. (by (a)) 1

(ii) $m_{OP} = \dfrac{k-0}{h-0} = \dfrac{k}{h}$, $m_{OR} = \dfrac{h-0}{k-0} = \dfrac{h}{k}$

Since $m_{OP}m_{OR} = 1 \neq -1$, hence NO. 1M + 1

(8)