SHAU KEI WAN GOVERNMENT SECONDARY SCHOOL

Yearly Examination 2019/20

Mathematics Paper 1

QUESTION–ANSWER BOOK

Sec 3(   )

Date : 2/7/2020
Time : 8:30 – 10:15 am
Max. mark : 100

Name : ________________________

Class No. : _________

Marks awarded : ________

Instructions:

1. The paper consists of 3 sections A, B and C.
2. Attempt ALL questions.
3. Write your answers in the QUESTION-ANSWER BOOK provided.
4. Unless otherwise specified, all working must be clearly shown.
5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
6. The diagrams in this paper are not necessarily drawn to scale.
7. The question marked with an asterisk (*) is challenging.

Section A (52 marks)

1. Simplify the following expressions and express the answers with positive indices.
   (*All the letters in the expressions represent non-zero numbers.*)

   (a) $(-x^0)^5$ (2 marks)

   (b) $\dfrac{x^{-10}y^5}{y^3}$ (2 marks)

2. Factorize

   (a) $3p^2q - 27q$, (2 marks)

   (b) $25x^2 + 10x + 1$. (2 marks)

3. Factorize

   (a) $a^2x - a^2y + a^3$, (1 mark)

   (b) $a^2x - a^2y + a^3 - x + y - a$. (3 marks)

4. Bank A offers an interest rate of 6% p.a. compounded half-yearly. Bank B offers an interest rate of 7% p.a. compounded yearly. Mr Tse wants to deposit \$100 000 in one of the two banks for 5 years. Which bank should he choose so that he can receive more interest? Explain your answer.

(4 marks)

5. Solve the inequality $\frac{3}{4} - x \leq \frac{3}{5} + \frac{x}{2}$ and represent the solutions graphically.

(4 marks)

6. It is given that the mean of the following set of data is 15.

4, x, 9, 25, 17, 27, 6, 20

(a) Find the value of x. (2 marks)

(b) Find the median and the mode of this set of data. (2 marks)

(a)

(b)

7. The following table shows the results of Cindy and Sally in different parts of a Putonghua examination and the weight assigned to each part.

	Phonetic transcription	**Listening**	**Oral**
Cindy	95	80	64
Sally	79	75	90
Weight	4	3	3

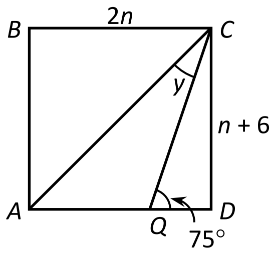
(a) Find the weighted mean marks of Cindy and Sally in the Putonghua examination. (3 marks)

(b) Whose performance is better? Explain your answer. (1 mark)

(a)

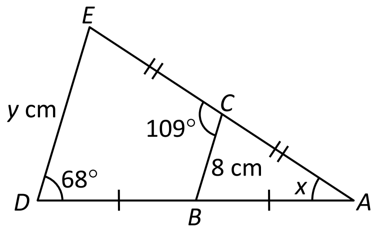
(b)

8. In the figure, $ABCD$ is a square. AQD is a straight line. Find the values of y and n.



(4 marks)

9. In the figure, ACE and ABD are straight lines. $AB = BD$ and $AC = CE$. Find the values of x and y.



(4 marks)

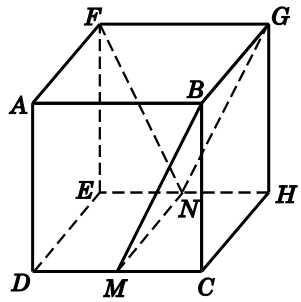
10. The figure shows a cube. M and N are the mid-points of DC and EH respectively.

(a) Find the projection of line segment AD on the plane $FEHG$.

(b) Name the angle between BM and the plane $DCHE$.

(c) Name the angle between FN and the plane $ADEF$.

(d) Name the angle between the planes $BGNM$ and $BGHC$.



(4 marks)

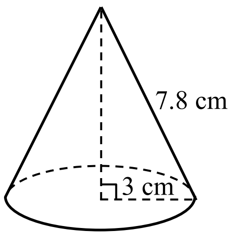
(a) ______________________________

(b) ______________________________

(c) ______________________________

(d) ______________________________

11.



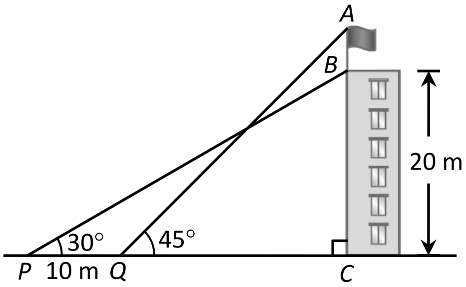
The figure shows a right circular cone of slant height 7.8 cm and base radius 3 cm.

(a) Find the total surface area of the cone. (2 marks)

(b) Find the volume of the cone. (2 marks)

(*Give the answers in terms of* π.)

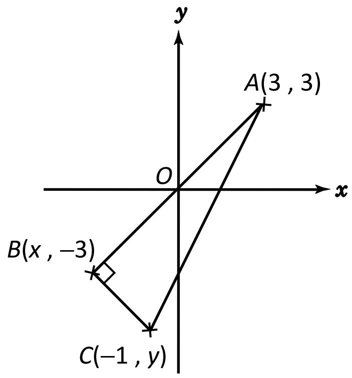
12.



In the figure, AB is a flagpole standing on the building BC and ABC is a straight line. The height of the building BC is 20 m. The angle of elevation of A from Q is 45°, while the angle of elevation of B from P is 30°. P and Q are 10 m apart, and PQC is a horizontal line. Find the height of the flagpole.
(*Give the answer correct to 3 significant figures.*)

(4 marks)

13.



Refer to the figure. *AOB* is a straight line. The slope of the line AB is 1. Find the values of *x* and *y*.

(4 marks)

Section B (38 marks)

14. A set of data 14, 36, *m*, 23, *n* and 23 is given. It is known that the mode of the set of data is 36.

(a) Find the median of the set of data. (3 marks)

(b) If the set of data is first doubled and then increased by 5, find the new median. (2 marks)

(a)

(b)

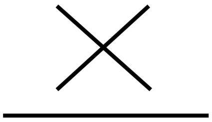
15. Factorize

(a) $p^2 - 6p - 16$, (2 marks)

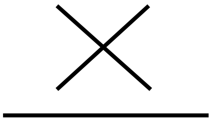
(b) $2b^2 - 7ab - 4a^2$, (2 marks)

(c) $128 + 2b^3$. (2 marks)

(a)



(b)



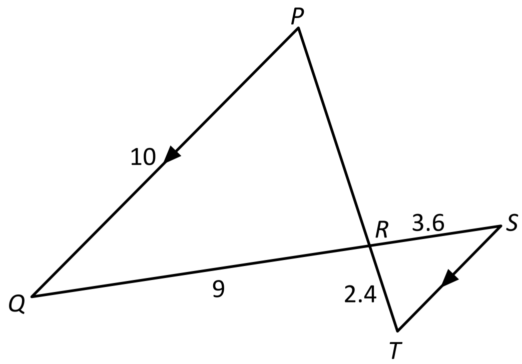
(c)

16. (a) Convert the denary number $1\ 117_{10}$ into a binary number. (3 marks)

(b) Convert the denary number $5\ 227_{10}$ into a hexademical number. (3 marks)

(a) (b)

17.



In the figure, PQ // ST. PT and QS intersect at R. $PQ = 10$, $QR = 9$, $RS = 3.6$ and $RT = 2.4$.

(a) Prove that $\triangle PQR \sim \triangle TSR$. (4 marks)

(b) Find the perimeter of $\triangle PQR$. (3 marks)

(a)

(b)

18. The length, width and height of a cuboid are 15 cm, 12 cm and 18 cm respectively. If the length increases by 20%, the width decreases by 25%, and the volume increases by 8%, find

(a) the new volume, (2 marks)

(b) the new height, (3 marks)

(c) the percentage change in the height of the cuboid. (2 marks)

(a)

(b)

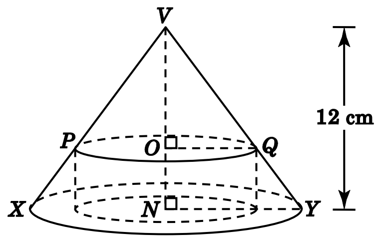
(c)

19. Alex has 30 coins. Each of the coins is either a \$2 coin or a \$5 coin. The total value of coins is not greater than \$100. At most how many \$5 coins does he have?

(7 marks)

Section C: Challenging Question (10 marks)

*20.



In the figure, a cylinder is inscribed in a right circular cone *VXY*, whose height *VN* is 12 cm. The volume of the cylinder is 1.5 times that of the cone *VPQ*.

(a) Find the height *ON* of the cylinder. (3 marks)

(b) Find the ratio of the base radius of the cylinder to that of the cone *VXY*. (3 marks)

(c) It is known that the ratio of the curved surface area of the cone *VXY* to that of the cylinder is 45 : 16. Find the total surface area of the cone *VXY* in terms of π. (4 marks)

(a)

(b)

(c)

-End of Paper-

SHAU KEI WAN GOVERNMENT SECONDARY SCHOOL

Yearly Examination 2019/20

Mathematics Paper 1

<u>Marking Scheme</u>

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7. The question marked with an asterisk (*) is challenging.

Section A (52 marks)

1. Simplify the following expressions and express the answers with positive indices.
(*All the letters in the expressions represent non-zero numbers.*)

(a) $(-x^0)^5$ (2 marks)

(b) $\dfrac{x^{-10}y^5}{y^3}$ (2 marks)

(a) $(-x^0)^5 = (-1)^5$
$= \underline{\underline{-1}}$

(b) $\dfrac{x^{-10}y^5}{y^3} = \dfrac{y^{5-3}}{x^{10}}$
$= \underline{\underline{\dfrac{y^2}{x^{10}}}}$

2. Factorize

(a) $3p^2q - 27q$, (2 marks)

(b) $25x^2 + 10x + 1$. (2 marks)

(a)
$3p^2q - 27q = 3q(p^2 - 9)$
$= 3q(p^2 - 3^2)$
$= \underline{\underline{3q(p+3)(p-3)}}$

(b)
$25x^2 + 10x + 1 = (5x)^2 + 2(5x)(1) + 1^2$
$= \underline{\underline{(5x+1)^2}}$

3. Factorize

(a) $a^2x - a^2y + a^3$, (1 mark)

(b) $a^2x - a^2y + a^3 - x + y - a$. (3 marks)

(a)
$a^2x - a^2y + a^3 = \underline{\underline{a^2(x - y + a)}}$

(b)
$a^2x - a^2y + a^3 - x + y - a$
$= a^2(x - y + a) - (x - y + a)$
$= (a^2 - 1)(x - y + a)$
$= (a^2 - 1^2)(x - y + a)$
$= \underline{\underline{(a+1)(a-1)(x-y+a)}}$

4. Bank *A* offers an interest rate of 6% p.a. compounded half-yearly. Bank *B* offers an interest rate of 7% p.a. compounded yearly. Mr Tse wants to deposit \$100 000 in one of the two banks for 5 years. Which bank should he choose so that he can receive more interest? Explain your answer.

(4 marks)

For bank *A*,

interest rate for half a year = $\frac{6\%}{2}$ = 3%

Taking half a year as a period,

number of periods in 5 years = 5 × 2 = 10

interest = \$100 000 × [$(1 + 3\%)^{10}$ – 1]

= \$34 392, *cor. to the nearest dollar*

For bank *B*,

interest = \$100 000 × [$(1 + 7\%)^{5}$ – 1]

= \$40 255, *cor. to the nearest dollar*

∵ \$40 255 > \$34 392

∴ He should choose bank *B*.

5. Solve the inequality $\frac{3}{4} - x \le \frac{3}{5} + \frac{x}{2}$ and represent the solutions graphically.

(4 marks)

$$\frac{3}{4} - x \le \frac{3}{5} + \frac{x}{2}$$

$$20\left(\frac{3}{4} - x\right) \le 20\left(\frac{3}{5} + \frac{x}{2}\right)$$

$$15 - 20x \le 12 + 10x$$

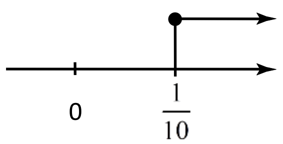
$$-20x - 10x \le 12 - 15$$

$$-30x \le -3$$

$$x \ge \frac{-3}{-30}$$

$$x \ge \frac{1}{10}$$

Graphical representation:



6. It is given that the mean of the following set of data is 15.

4, x, 9, 25, 17, 27, 6, 20

(a) Find the value of x. (2 marks)

(b) Find the median and the mode of this set of data. (2 marks)

(a) $15 = \frac{4+x+9+25+17+27+6+20}{8}$

$15 \times 8 = 108 + x$

$x = 120 - 108$

$= \underline{\underline{12}}$

(b) Arranging the data in ascending order:

4, 6, 9, 12, 17, 20, 25, 27

Median $= \frac{12+17}{2}$

$= \underline{\underline{14.5}}$

There is no mode in this set of data.

7. The following table shows the results of Cindy and Sally in different parts of a Putonghua examination and the weight assigned to each part.

	Phonetic transcription	**Listening**	**Oral**
Cindy	95	80	64
Sally	79	75	90
Weight	4	3	3

(a) Find the weighted mean marks of Cindy and Sally in the Putonghua examination. (3 marks)

(b) Whose performance is the best? Explain your answer. (1 mark)

(a) Weighted mean mark of Cindy

$= \frac{95\times4+80\times3+64\times3}{4+3+3}$

$= \underline{\underline{81.2}}$

Weighted mean mark of Sally

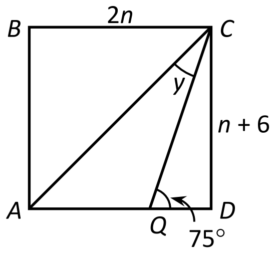
$= \frac{79\times4+75\times3+90\times3}{4+3+3}$

$= \underline{\underline{81.1}}$

(b) $\because$ 81.2 > 81.1

$\therefore$ Cindy's performance is the best.

8. In the figure, *ABCD* is a square. *AQD* is a straight line. Find the values of y and n.



(4 marks)

In $\triangle ACQ$,

$\angle CAQ = 45°$ (*property of square*)

$y + \angle CAQ = \angle CQD$ (*ext.* $\angle$ *of* $\triangle$)

$y + 45° = 75°$

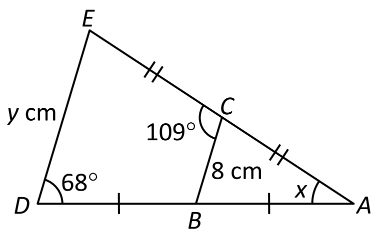
$y = \underline{\underline{30°}}$

$BC = CD$ (*by definition*)

$2n = n + 6$

$n = \underline{\underline{6}}$

9. In the figure, *ACE* and *ABD* are straight lines. $AB = BD$ and $AC = CE$. Find the values of x and y.



(4 marks)

In $\triangle AED$,

$\because$ $AB = BD$ and $AC = CE$ (*given*)

$\therefore$ $BC = \frac{1}{2}DE$ and $BC \parallel DE$ (*mid-pt. theorem*)

$8 = \frac{1}{2}y$

$y = \underline{\underline{16}}$

$\angle ABC = \angle ADE = 68°$ (*corr.* $\angle$*s,* $BC \parallel DE$)

In $\triangle ABC$,

$\angle BAC + \angle ABC = \angle BCE$ (*ext.* $\angle$ *of* $\triangle$)

$x + 68° = 109°$

$x = \underline{\underline{41°}}$

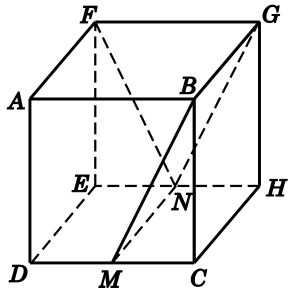
10. The figure shows a cube. *M* and *N* are the mid-points of *DC* and *EH* respectively.

(a) Find the projection of line segment AD on the plane $FEHG$.

(b) Name the angle between BM and the plane $DCHE$.

(c) Name the angle between FN and the plane $ADEF$.

(d) Name the angle between the planes $BGNM$ and $BGHC$.



(4 marks)

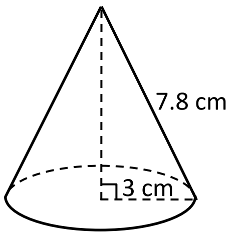
(a) The projection of line segment AD on the plane $FEHG$ is FE. 1A

(b) The angle between BM and the plane $DCHE$ is $\angle BMC$. 1A

(c) The angle between FN and the plane $ADEF$ is $\angle NFE$. 1A

(d) The angle between the planes $BGNM$ and $BGHC$ is $\angle MBC$ (or $\angle NGH$). 1A

11.



The figure shows a right circular cone of slant height 7.8 cm and base radius 3 cm.

(a) Find the total surface area of the cone. (2 marks)

(b) Find the volume of the cone. (2 marks)

(*Give the answers in terms of* π.)

(a) Total surface area of the cone $= (\pi \times 3 \times 7.8 + \pi \times 3^2)\text{ cm}^2$

$= \underline{\underline{32.4\pi\text{ cm}^2}}$

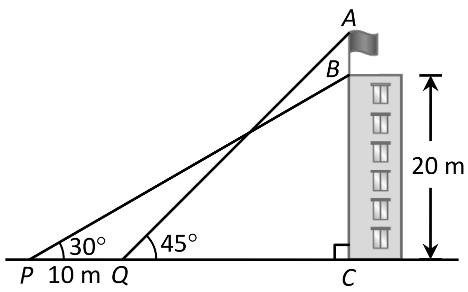
(b) Height of the cone $= \sqrt{7.8^2 - 3^2}$ cm

$= 7.2$ cm

$\therefore$ Volume of the cone $= \dfrac{1}{3} \times \pi \times 3^2 \times 7.2\ \text{cm}^3$

$= \underline{\underline{21.6\pi\text{ cm}^3}}$

12.



In the figure, AB is a flagpole standing on the building BC and ABC is a straight line. The height of the building BC is 20 m. The angle of elevation of A from Q is 45°, while the angle of elevation of B from P is 30°. P and Q are 10 m apart, and PQC is a horizontal line. Find the height of the flagpole.
(*Give the answer correct to 3 significant figures.*)

(4 marks)

In $\triangle BCP$,

$$\tan 30° = \frac{20 \text{ m}}{PC}$$

$$PC = \frac{20}{\tan 30°} \text{ m}$$

$$QC = PC - PQ$$

$$= \left(\frac{20}{\tan 30°} - 10\right) \text{ m}$$

In $\triangle ACQ$,

$$\tan 45° = \frac{AC}{QC}$$

$$AC = QC \tan 45°$$

$$= \left(\frac{20}{\tan 30°} - 10\right) \tan 45° \text{ m}$$

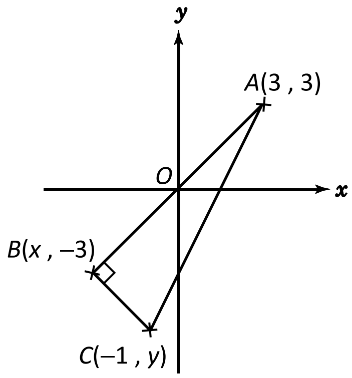
$$AB = AC - BC$$

$$= \left[\left(\frac{20}{\tan 30°} - 10\right) \tan 45° - 20\right] \text{ m}$$

= 4.64 m, *cor. to 3 sig. fig.*

∴ The height of the flagpole is 4.64 m.

13.



Refer to the figure. *AOB* is a straight line. The slope of the line AB is 1. Find the values of *x* and *y*.

(4 marks)

$$\frac{3-(-3)}{3-x}=1$$

$$\frac{6}{3-x}=1$$

$$6=3-x$$

$$x=\underline{\underline{-3}}$$

$\because$ $AB \perp BC$

$\therefore$ Slope of AB × slope of $BC = -1$

$$1\times\frac{y-(-3)}{-1-(-3)}=-1$$

$$\frac{y+3}{2}=-1$$

$$y+3=-2$$

$$y=\underline{\underline{-5}}$$

Section B (38 marks)

14. A set of data 14, 36, *m*, 23, *n* and 23 is given. It is known that the mode of the set of data is 36.

(a) Find the median of the set of data. (3 marks)

(b) If the set of data is first doubled and then increased by 5, find the new median. (2 marks)

(a) Since the datum 23 appears twice and the mode is 36, the frequency of the datum 36 must be more than two.

$\therefore$ *m* and *n* must be equal to 36.

Arranging the data in ascending order:

14, 23, 23, 36, 36, 36

Median = $\frac{23+36}{2}$ = $\underline{\underline{29.5}}$

(b) New median = 29.5 × 2 + 5

= $\underline{\underline{64}}$

15. Factorize

(a) $p^2 - 6p - 16$, (2 marks)

(b) $2b^2 - 7ab - 4a^2$, (2 marks)

(c) $128 + 2b^3$. (2 marks)

(a)

$p \quad\quad 2$

$p \quad\quad -8$

$2p \quad\quad -8p = -6p$

$p^2 - 6p - 16 = \underline{\underline{(p + 2)(p - 8)}}$

(b)

$2b \quad\quad a$

$b \quad\quad -4a$

$ab \quad\quad -8ab = -7ab$

$2b^2 - 7ab - 4a^2 = \underline{\underline{(2b + a)(b - 4a)}}$

(c)

$$\begin{aligned} 128 + 2b^3 &= 2(64 + b^3) \\ &= 2(4^3 + b^3) \\ &= 2(4 + b)[4^2 - (4)(b) + b^2] \\ &= \underline{\underline{2(4 + b)(16 - 4b + b^2)}} \end{aligned}$$

16. (a) Convert the denary number $1\ 117_{10}$ into a binary number. (3 marks)

(b) Convert the denary number $5\ 227_{10}$ into a hexademical number. (3 marks)

(a)

2) 1 117
2) 558 ⋯⋯ 1
2) 279 ⋯⋯ 0
2) 139 ⋯⋯ 1
2) 69 ⋯⋯ 1
2) 34 ⋯⋯ 1
2) 17 ⋯⋯ 0
2) 8 ⋯⋯ 1
2) 4 ⋯⋯ 0
2) 2 ⋯⋯ 0
1 ⋯⋯ 0

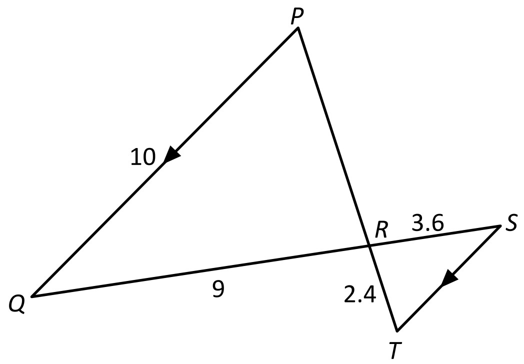
$\therefore \quad 1\ 117_{10} = \underline{\underline{10001011101_2}}$

(b)

16) 5 227
16) 326 ⋯⋯ 11
16) 20 ⋯⋯ 6
1 ⋯⋯ 4

$\therefore \quad 5\ 227_{10} = \underline{\underline{146B_{16}}}$

17.



In the figure, $PQ // ST$. PT and QS intersect at R. $PQ = 10$, $QR = 9$, $RS = 3.6$ and $RT = 2.4$.

(a) Prove that $\triangle PQR \sim \triangle TSR$. (4 marks)

(b) Find the perimeter of $\triangle PQR$. (3 marks)

(a) In $\triangle PQR$ and $\triangle TSR$,

$\angle PQR = \angle TSR$	*alt.* $\angle s$, $PQ // ST$
$\angle QPR = \angle STR$	*alt.* $\angle s$, $PQ // ST$
$\angle PRQ = \angle TRS$	*vert. opp.* $\angle s$
$\therefore$ $\underline{\underline{\triangle PQR \sim \triangle TSR}}$	*AAA*

(b) $\because$ $\triangle PQR \sim \triangle TSR$ (*proved in* (a))

$\therefore$ $\dfrac{PR}{TR} = \dfrac{QR}{SR}$ (*corr. sides,* $\sim \triangle s$)

$\dfrac{PR}{2.4} = \dfrac{9}{3.6}$

$PR = 6$

Perimeter of $\triangle PQR = PQ + QR + PR$

$= 10 + 9 + 6$

$= \underline{\underline{25}}$

18. The length, width and height of a cuboid are 15 cm, 12 cm and 18 cm respectively. If the length increases by 20%, the width decreases by 25%, and the volume increases by 8%, find
  (a) the new volume, (2 marks)
  (b) the new height, (3 marks)
  (c) the percentage change in the height of the cuboid. (2 marks)

(a) Original volume of the cuboid $= 15 \times 12 \times 18 \text{ cm}^3$

$= 3\ 240 \text{ cm}^3$

New volume of the cuboid $= 3\ 240 \times (1 + 8\%) \text{ cm}^3$

$= \underline{\underline{3\ 499.2 \text{ cm}^3}}$

(b) New length of the cuboid $= 15 \times (1 + 20\%) \text{ cm}$

$= 18 \text{ cm}$

New width of the cuboid $= 12 \times (1 - 25\%) \text{ cm}$

$= 9 \text{ cm}$

Let h cm be the new height of the cuboid.

$$18 \times 9 \times h = 3\ 499.2$$

$$h = 21.6$$

∴ The new height of the cuboid is 21.6 cm.

(c) Percentage change in the height of the cuboid

$= \dfrac{21.6 - 18}{18} \times 100\%$

$= \underline{\underline{+20\%}}$

19. Alex has 30 coins. Each of the coins is either a \$2 coin or a \$5 coin. The total value of coins is not greater than \$100. At most how many \$5 coins does he have?

(7 marks)

Let x be the number of \$5 coins.

Then, the number of \$2 coins is $30 - x$.

$$5x + 2(30 - x) \le 100$$
$$3x + 60 \le 100$$
$$3x \le 40$$
$$x \le 13\frac{1}{3}$$

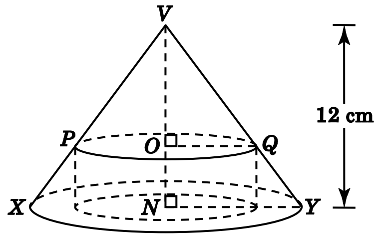
$\because$ x is an integer.

$\therefore$ The maximum value of x is 13.

$\therefore$ <u>Alex has at most 13 \$5 coins.</u>

Section C: Challenging Question (10 marks)

*20.



In the figure, a cylinder is inscribed in a right circular cone *VXY*, whose height *VN* is 12 cm. The volume of the cylinder is 1.5 times that of the cone *VPQ*.

(a) Find the height *ON* of the cylinder. (3 marks)

(b) Find the ratio of the base radius of the cylinder to that of the cone *VXY*. (3 marks)

(c) It is known that the ratio of the curved surface area of the cone *VXY* to that of the cylinder is 45 : 16. Find the total surface area of the cone *VXY* in terms of π. (4 marks)

(a) Let r cm and h cm be the base radius and height of the cylinder respectively.

Then $VO = VN - ON = (12 - h)$ cm.

$$\pi r^2 h = 1.5 \times \frac{1}{3}\pi r^2 (12 - h)$$

$$2h = 12 - h$$

$$3h = 12$$

$$h = 4$$

∴ <u>The height *ON* of the cylinder is 4 cm.</u>

(b) Let R cm be the base radius of the cone *VXY*.

∵ $\triangle VOQ \sim \triangle VNY$ (*AAA*)

∴ $\frac{OQ}{NY} = \frac{VO}{VN}$

$$\frac{r}{R} = \frac{12 - 4}{12}$$

$$= \frac{2}{3}$$

∴ The required ratio = <u>2 : 3</u>

(c) Let ℓ cm be the slant height of the cone *VXY*.

$$\frac{\pi R \ell}{2\pi r \times 4} = \frac{45}{16}$$

$$\frac{\ell}{8} \times \frac{3}{2} = \frac{45}{16}$$

$$\ell = 15$$

i.e. $VY = 15$ cm

In $\triangle VNY$, $NY = \sqrt{VY^2 - VN^2} = \sqrt{15^2 - 12^2}$ cm $= 9$ cm

∴ The required total surface area $= (\pi \times 9 \times 15 + \pi \times 9^2)$ cm^2

$= \underline{\underline{216\pi \text{ cm}^2}}$

-End of Paper-