

# MATHEMATICS

## PAPER 1

Secondary 3

Date: 10 – 01 – 2011

Time: 8:30 am – 10:00 am



- Write your class, class number in the spaces provided on this cover.
- This paper consists of TWO sections, A and B. Section A and Section B carry 80 marks and 40 marks respectively.
- Attempt ALL questions in this paper. Write your answer in the spaces provided in this Question-Answer Book.
- Unless otherwise specified, all working must be clearly shown.
- Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.
- The diagrams in this paper are not necessarily drawn to scale.

Class	
Class Number	

Question No.	Teacher's Use Only	
	Max. Marks	Marks
<b>Section A</b>		
1	10	
2	6	
3	10	
4	6	
5	8	
6	8	
7	8	
8	9	
9	7	
10	8	
<b>Section B</b>		
11	20	
12	20	
<b>Total</b>	120	

**SECTION A Short questions. (80 marks)**

1 (a) Factorize  $x^2 - 9$

(2 marks)

(b) Factorize  $x^3 - 27$

(2 marks)

(c) Using the results in (a) and (b), factorize  $2x^6 - 1458$

(6 marks)

2 Factorize  $(7x^2 - 5x + 1) + (5x^2 - 2x + 7) - (4x^2 + 19x + 2)$

(6 marks)

- 3 (a) Without using calculator, simplify  $4^0 + 4^{-1} + 4^{-2}$

(4 marks)

(b) Simplify  $\frac{4 \times 5^n - 25 \times 5^{n-2}}{5^n + 5^{n+1}}$

(6 marks)

4 Express  $3.\dot{2}1\dot{4}$  in the form  $\frac{a}{b}$ , where  $a, b$  are integers and  $b > 0$ .

(6 marks)

5 Simplify  $\sqrt[5]{(p+q)^7} \times (p+q)^{\frac{-2}{5}} \times \frac{1}{p+q}$ .

(8 marks)

- 6 (a) Jack sold a piece of land to Tommy at a profit of 20%. Tommy sold it to Mary at a loss of 30%. Mary sold it to Johnson for  $\$2.1 \times 10^9$  at a profit of 25%. Find the profit that Jack made. (8 marks)

7. Solve  $\frac{5+3x}{2} - 1 < \frac{2x+3}{3} + x$ , and represent the solution graphically (8 marks)

8 The following table shows the frequency distribution of fire incidents in a district in a period of 60 days.

No. of incidents in one day	0	1	2	3	4	5	6	7	8 or more
No. of days	12	15	13	12	5	2	0	1	0

Find the probability that

(a) there is no incident in a day.

(3 marks)

(b) there are exactly 4 fire incidents in a day, and

(3 marks)

(c) there are at least 4 fire incidents in a day.

(3 marks)

- 9 Evaluate  $\frac{0.000000000512 \times 6\,000\,000000}{0.000\,000\,000\,00012}$  and express the answer in scientific notation.

(7 marks)

- 10 The salaries of the staffs in an office are given below:  
\$3000, \$4000, \$3000, \$9000, \$4000, \$5000, \$4000, \$66000

(a) Find the mean, mode and median.

(6 marks)

(b) Explain briefly why the mean is not a good way to represent the data set. (2 marks)

**SECTION B Long questions (40 marks)**

11. The table below shows the marks of 100 S.3 students in a Mathematics form test.

Marks	Class Mark	Frequency
41 – 50	45.5	11
51 – 60	55.5	21
61 – 70	65.5	23
71 – 80	75.5	25
81 – 90	85.5	12
91 – 100	95.5	8

(a) Complete the following cumulative frequency table.

(2 marks)

Marks less than	Cumulative frequency
40.5	
50.5	
60.5	
70.5	
80.5	
90.5	
100.5	

(b) Draw the cumulative frequency polygon.

(4 marks)

- (c) Find the mean marks of the S.3 students ( 2 marks )
- (d) Find the median of the S.3 students. ( 2 marks )
- (e) Find the modal class of the S.3 students ( 1 mark )
- (f) Let the full mark for the test be 141. The student who obtains more than 70.5 marks get a pass in the test.
- (i) Find the passing percentage of the S.3 students. ( 4 marks )
- (ii) (ii) Since the passing percentage is too low, the teachers decide to add 10 marks to each student. Find the new mean, the new median and the new modal class of the marks after the adjustment. ( 3 marks )
- (iii) The school will offer remedial class for the students who failed the test. How many students would be chosen to the remedial class? ( 2 marks )

- 12 Mr. Chan is 55 years old and his daughter is in Secondary 3. He is an account clerk. He had deposited \$12000 per year to the MPF since he was 50. His employer promises to pay the same amount to the MPF. The MPF financial organization guarantees a return of 5% per annum. In addition, Mr. Chan bought a saving insurance for his daughter 3 years ago. He save \$50000 per year. The insurance company guarantees a return of 4% per annum. His daughter could get the sum of money when she goes to university.

(Mr. Chan deposits the MPF and saving insurance at the beginning of the year and gets back the money at the end of the year)

(Give the answer correct to the nearest \$100.)

- (a) When Mr. Chan's daughter finishes her Secondary school life, she wants to study abroad. How much money can she get for her university study? (10 marks)

- (b) The money for Mr. Chan's daughter is not sufficient for her to study abroad. Mr. Chan sponsors half of his MPF money to his daughter when he retires at 60. How much money does he leave behind for his expenses after retirement?

(10 marks)

**END OF PAPER**

# MATHEMATICS

## PAPER 1

### Answer

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11	20	
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<b>Total</b>	120	

## SECTION A Short questions. (80 marks)

- 1 (a) Factorize
- $x^2 - 9$
- (2 marks)

$$x^2 - 3^2$$

$$= (x+3)(x-3)$$

- (b) Factorize
- $x^3 - 27$
- (2 marks)

$$x^3 - 3^3$$

$$= (x-3)(x^2+3x+9)$$

- (c) Using the results in (a) and (b), factorize
- $2x^6 - 1458$
- (6 marks)

$$2x^6 - 1458$$

$$= 2[(x^3)^2 - (3^3)^2]$$

$$= 3(x^2+3^3)(x^3-3^3)$$

$$= 3(x+3)(x-3)(x^2-3x+9)(x^2+3x+9)$$

or

$$2x^6 - 1458$$

$$= 2[(x^2)^3 - (3^2)^3]$$

$$= 2(x^2-3^2)(x^4+9x^2+81)$$

$$= 2(x+3)(x-3)(x^4+18x^2+81-9x^2)$$

$$= 2(x+3)(x-3)[(x^2+9)^2 - (3x)^2]$$

$$= 2(x+3)(x-3)(x^2+3x+9)(x^2-3x+9)$$

- 2 Factorize
- $(7x^2 - 5x + 1) + (5x^2 - 2x + 7) - (4x^2 + 19x + 2)$
- (6 marks)

$$7x^2 - 5x + 1 + 5x^2 - 2x + 7 - 4x^2 - 19x - 2$$

$$= 7x^2 + 5x^2 - 4x^2 - 5x - 2x - 19x + 1 + 7 - 2$$

$$= 8x^2 - 26x + 6$$

$$= 2(4x^2 - 13x + 3)$$

$$= 2(4x-1)(x-3)$$

- 3 (a) Without using calculator, simplify  $4^0 + 4^{-1} + 4^{-2}$

(4 marks)

$$\begin{aligned} & 4^0 + 4^{-1} + 4^{-2} \\ &= 1 + \frac{1}{4} + \frac{1}{16} \\ &= \frac{16 + 4 + 1}{16} \\ &= \frac{21}{16} \end{aligned}$$

- (b) Simplify  $\frac{4 \times 5^n - 25 \times 5^{n-2}}{5^n + 5^{n+1}}$

(6 marks)

$$\begin{aligned} & \frac{4 \times 5^n - 5^2 \times 5^{n-2}}{5^n + 5 \times 5^n} \\ &= \frac{4 \times 5^n - 5^{n-2+2}}{6 \times 5^n} \\ &= \frac{4 \times 5^n - 5^n}{6 \times 5^n} \\ &= \frac{3 \times 5^n}{6 \times 5^n} \\ &= \frac{1}{2} \end{aligned}$$

- 4 Express  $3.\dot{2}1\dot{4}$  in the form  $\frac{a}{b}$ , where  $a, b$  are integers and  $b > 0$ .

(6 marks)

$$\text{Let } x \text{ be } 3.\dot{2}1\dot{4}$$

$$1000x = 3214.\dot{2}1\dot{4}$$

$$-) \quad x = 3.\dot{2}1\dot{4}$$

$$999x = 3211$$

$$x = \frac{3211}{999}$$

$$\therefore 3.\dot{2}1\dot{4} = \frac{3211}{999}$$

- 5 Simplify  $\sqrt[5]{(p+q)^7} \times (p+q)^{-\frac{2}{5}} \times \frac{1}{p+q}$ .

(8 marks)

$$(p+q)^{\frac{7}{5}} \times (p+q)^{-\frac{2}{5}} \times \frac{1}{p+q}$$

$$= (p+q)^{\frac{7}{5} + (-\frac{2}{5}) + (-1)}$$

$$= (p+q)^0$$

$$= 1$$

- 6 (a) Jack sold a piece of land to Tommy at a profit of 20%. Tommy sold it to Mary at a loss of 30%. Mary sold it to Johnson for  $\$2.1 \times 10^9$  at a profit of 25%. Find the profit that Jack made. (8 marks)

Let  $\$x$  be the cost of the land

$$x(1+20\%)(1-30\%)(1+25\%) = 2.1 \times 10^9$$

$$x = \frac{2.1 \times 10^9}{(1.2)(0.7)(1.25)}$$

$$x = 2 \times 10^9$$

$$\begin{aligned} \therefore \text{The profit that Jack made} &= 2 \times 10^9 \times 20\% \\ &= \$4 \times 10^8 \end{aligned}$$

7. Solve  $\frac{5+3x}{2} - 1 < \frac{2x+3}{3} + x$ , and represent the solution graphically (8 marks)

$$\frac{5+3x-2}{2} < \frac{2x+3+3x}{3}$$

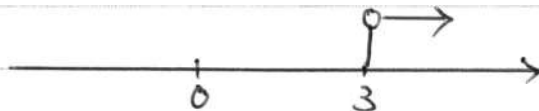
$$\frac{3+3x}{2} < \frac{5x+3}{3}$$

$$3(3+3x) < 2(5x+3)$$

$$9+9x < 10x+6$$

$$3 < x$$

$$\therefore x > 3$$



- 8 The following table shows the frequency distribution of fire incidents in a district in a period of 60 days.

No. of incidents in one day	0	1	2	3	4	5	6	7	8 or more
No. of days	12	15	13	12	5	2	0	1	0

Find the probability that

- (a) there is no incident in a day.

(3 marks)

$$P(0) = \frac{12}{60}$$

$$= \frac{1}{5}$$

- (b) there are exactly 4 fire incidents in a day, and

(3 marks)

$$P(4) = \frac{5}{60}$$

$$= \frac{1}{12}$$

- (c) there are at least 4 fire incidents in a day.

(3 marks)

$$P(\text{at least } 4) = \frac{5+2+1}{60}$$

$$= \frac{8}{60}$$

$$= \frac{2}{15}$$

- 9 Evaluate  $\frac{0.000000000512 \times 6\,000\,000\,000}{0.000\,000\,000\,00012}$  and express the answer in scientific notation. (7 marks)

$$\frac{5.12 \times 10^{-10} \times 6 \times 10^9}{1.2 \times 10^{-13}}$$

$$= \frac{5.12 \times 6}{1.2} \times 10^{-10+9-(-13)}$$

$$= 25.6 \times 10^{12}$$

$$= 2.56 \times 10^{13}$$

- 10 The salaries of the staffs in an office are given below:  
\$3000, \$4000, \$3000, \$9000, \$4000, \$5000, \$4000, \$66000

(a) Find the mean, mode and median.

(6 marks)

$$\text{Mean} = \frac{3000 \times 2 + 4000 \times 3 + 5000 + 9000 + 66000}{8}$$

$$= \$ 12250.$$

3000, 3000, 4000, 4000, 4000, 5000, 9000, 66000

$$\text{Median} = \$ 4000$$

$$\text{Mode} = \$ 4000$$

(b) Explain briefly why the mean is not a good way to represent the data set. (2 marks)

There is an extremely large data in the data set.

**SECTION B Long questions (40 marks)**

11. The table below shows the marks of 100 S.3 students in a Mathematics form test.

Marks	Class Mark	Frequency
41 – 50	45.5	11
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71 – 80	75.5	25
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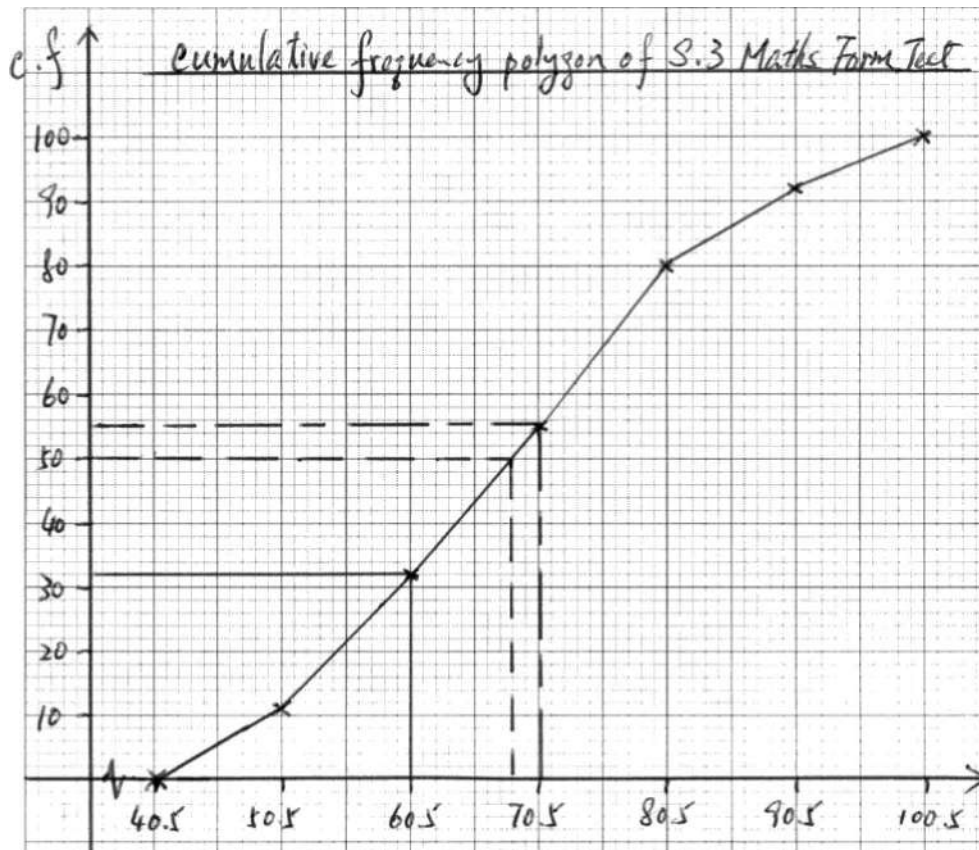
(a) Complete the following cumulative frequency table.

(2 marks)

Marks less than	Cumulative frequency
40.5	0
50.5	11
60.5	32
70.5	55
80.5	80
90.5	92
100.5	100

(b) Draw the cumulative frequency polygon.

(4 marks)



- (c) Find the mean marks of the S.3 students (2 marks)

$$\frac{45.5 \times 11 + 55.5 \times 21 + 65.5 \times 23 + 75.5 \times 25 + 85.5 \times 12 + 95.5 \times 8}{100}$$

$$= 68.5$$

- (d) Find the median of the S.3 students. (2 marks)

From graph, median = 68 (67.5 - 68.5)

- (e) Find the modal class of the S.3 students (1 mark)

The modal class = 71 - 80

- (f) Let the full mark for the test be 141. The student who obtains more than 70.5 marks get a pass in the test.

- (i) Find the passing percentage of the S.3 students. (4 marks)

From the graph, 55 students obtain 70.5 marks or below.

$$\text{Number of students pass} = 100 - 55$$

$$= 45$$

$$\therefore \text{Passing percentage} = \frac{45}{100} \times 100\%$$

$$= 45\%$$

- (ii) (ii) Since the passing percentage is too low, the teachers decide to add 10 marks to each student. Find the new mean, the new median and the new modal class of the marks after the adjustment. (3 marks)

$$\text{New mean} = 68.5 + 10 = 78.5$$

$$\text{New median} = 68 + 10 = 78$$

$$\text{New mode} = 81 - 90$$

- (iii) The school will offer remedial class for the students who failed the test. How many students would be chosen to the remedial class? (2 marks)

$$\text{New passing marks} = 60.5$$

$\therefore$  From the graph, 32 students would be chosen

12

Mr. Chan is 55 years old and his daughter is in Secondary 3. He is an account clerk. He had deposited \$12000 per year to the MPF since he was 50. His employer promises to pay the same amount to the MPF. The MPF financial organization guarantees a return of 5% per annum. In addition, Mr. Chan bought a saving insurance for his daughter 3 years ago. He save \$50000 per year. The insurance company guarantees a return of 4% per annum. His daughter could get the sum of money when she goes to university.

(Mr. Chan deposits the MPF and saving insurance at the beginning of the year and gets back the money at the end of the year)

(Give the answer correct to the nearest \$100.)

- (a) When Mr. Chan's daughter finishes her Secondary school life, she wants to study abroad. How much money can she get for her university study? (10 marks)

For the 1<sup>st</sup> \$5000, she gets back at the end of the 6 years

$$50000 (1+4\%)^6 = 63266$$

For the 2<sup>nd</sup> \$5000, she gets back

$$50000 (1+4\%)^5 = 60833$$

For the 3<sup>rd</sup> \$5000, she gets back

$$50000 (1+4\%)^4 = 58493$$

For the 4<sup>th</sup> \$5000, she gets back

$$50000 (1+4\%)^3 = 56243$$

For the 5<sup>th</sup> \$5000, she gets back

$$50000 (1+4\%)^2 = 54080$$

For the 6<sup>th</sup> \$5000, she gets back

$$50000 (1+4\%) = 52000$$

∴ Total money she can get

$$63266 + 60833 + 58493 + 56243 + 54080 + 52000$$

$$= 344900 \text{ (corr. to the nearest 100)}$$

- (b) The money for Mr. Chan's daughter is not sufficient for her to study abroad. Mr. Chan sponsors half of his MPF money to his daughter when he retires at 60. How much money does he leave behind for his expenses after retirement?

The total MPF

(10 marks)

$$24000 (1+5\%)^{10} = 39093$$

$$24000 (1+5\%)^9 = 37232$$

$$24000 (1+5\%)^8 = 35459$$

$$24000 (1+5\%)^7 = 33770$$

$$24000 (1+5\%)^6 = 32162$$

$$24000 (1+5\%)^5 = 30631$$

$$24000 (1+5\%)^4 = 29172$$

$$24000 (1+5\%)^3 = 27783$$

$$24000 (1+5\%)^2 = 26480$$

$$\text{+)} \quad 24000 (1+5\%) = 25200$$

$$\hline = 316962$$

$$\therefore \text{ He has } \$ \frac{316962}{2}$$

$$= 158500 \quad (\text{corr. to the nearest } \$100)$$

for his expenses after retirement.

**END OF PAPER**