

ST. STEPHEN'S GIRLS' COLLEGE

Mid-Year Examination 2020 – 2021

Form 3

155 students

WYL, SCHL, CYN, YLN

Mathematics

Time allowed: 1 hour 30 minutes

Question/Answer Paper

Please read the following instructions very carefully.

1. This paper consists of TWO sections, A and B.
2. Write your class, class number and name in the spaces provided on this cover.
3. This paper carries 100 marks. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question/Answer Paper.
4. The diagrams in this paper are not necessarily drawn to scale.
5. Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.

Class	
Class No.	
Name	

For Markers' Use Only		
1 – 16		(40)
17 – 18	(3)	(4)
19 – 20	(4)	(4)
21 – 22	(4)	(6)
23		(5)
24		(10)
25		(6)
26		(6)
27		(8)
TOTAL		(100)

Section A (40%)

All rough work should be done on the rough work paper provided, but will not be marked.

Questions		Answer	Marks
1.	Factorize the following polynomials. (a) $x^2 - 7x - 30$ (b) $-2p^2 + 50q^2$ (c) $2y^3 - 24y^2 + 72y$	1. (a) _____ (b) _____ (c) _____	1 2 2
2.	Determine which of the following statements is/are true. Circle the correct answer. (a) $144y^2 - 120xy + 25x^2 \equiv (5x - 12y)^2$ (b) $-a^2 + 6a - 9 \equiv (3 - a)^2$ (c) $64x^3 - 125 \equiv (4x - 5)(16x^2 + 20x + 25)$	2. (a) True / False (b) True / False (c) True / False	1 1 1
3.	Make b the subject of $y = a - \frac{c+b}{b}$.	3. _____	2
4.	If $\frac{2^{2n} \times 9^n}{3^n} = a^n$, where a is an integer, find a .	4. _____	2
5.	Express the following numbers in scientific notation. (a) $-2\,530\,000$ (b) $0.000\,000\,39$	5. (a) _____ (b) _____	1 1
6.	Arrange the following numbers in ascending order. I. -2.34×10^{-70} II. 2.34×10^{70} III. -2.34×10^{70} IV. 2.34×10^{-90}	6. _____ < _____ < _____ < _____	2
7.	Consider the binary number $10\underline{1}100_{(2)}$. (a) Write down the place value of the underlined digit. (b) Express $101100_{(2)}$ in the expanded form.	7. (a) _____	1
	Answer for (b) : _____		1
8.	Convert the decimal number $8^4 + 8^{11}$ into a hexadecimal number.	8. _____	2
9.	If \$32 000 is deposited in a bank at a simple interest rate of 3% p.a., how many years will it take to receive an amount of \$34 880?	9. _____	2

21. The value of a pair of earphones was \$800 in 2015 and its value has decreased at a fixed rate each year. In 2017, the value of the earphones decreased to \$648.
- (a) Find the decay factor of the value of the earphones. (2 marks)
 - (b) Suppose the decay factor of the value of the earphones remains unchanged, find the value of the earphones in 2011. Give your answer correct to the nearest dollar. (2 marks)

22. A carbon dioxide molecule consists of one carbon atom and two oxygen atoms. The weights of a carbon atom and an oxygen atom are 1.99×10^{-26} kg and 2.67×10^{-26} kg respectively. **(Express the answers of (a) and (b)(i) in scientific notation.)**
- (a) Find the weight of a carbon dioxide molecule. (2 marks)
 - (b) A carbon dioxide extinguisher contains 1.9791 kg of carbon dioxide.
 - (i) Find the number of carbon dioxide molecules in the extinguisher.
 - (ii) Find the total weight of the oxygen atoms in the extinguisher. (4 marks)

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SOLUTION

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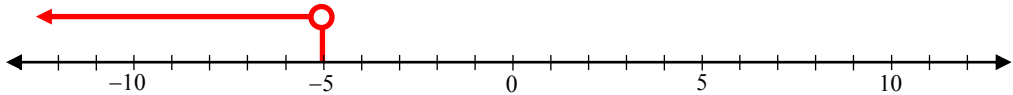
Class	
Class No.	
Name	

For Markers' Use Only		
1 – 16	(40)	
17 – 18	(3)	(4)
19 – 20	(4)	(4)
21 – 22	(4)	(6)
23	(5)	
24	(10)	
25	(6)	
26	(6)	
27	(8)	
TOTAL	(100)	

Section A (40%)

All rough work should be done on the rough work paper provided, but will not be marked.

Questions		Answer	Marks
1.	Factorize the following polynomials. (a) $x^2 - 7x - 30$ (b) $-2p^2 + 50q^2$ (c) $2y^3 - 24y^2 + 72y$	1. (a) $(x-10)(x+3)$ (b) $-2(p+5q)(p-5q)$ (c) $2y(y-6)^2$	1 2 2
2.	Determine which of the following statements is/are true. Circle the correct answer. (a) $144y^2 - 120xy + 25x^2 \equiv (5x - 12y)^2$ (b) $-a^2 + 6a - 9 \equiv (3 - a)^2$ (c) $64x^3 - 125 \equiv (4x - 5)(16x^2 + 20x + 25)$	2. (a) True / False (b) True / False (c) True / False	1 1 1
3.	Make b the subject of $y = a - \frac{c+b}{b}$.	3. $b = \frac{c}{a-1-y}$	2
4.	If $\frac{2^{2n} \times 9^n}{3^n} = a^n$, where a is an integer, find a .	4. 12	2
5.	Express the following numbers in scientific notation. (a) -2 530 000 (b) 0.000 000 39	5. (a) -2.53×10^6 (b) 3.9×10^{-7}	1 1
6.	Arrange the following numbers in ascending order. I. -2.34×10^{-70} II. 2.34×10^{70} III. -2.34×10^{70} IV. 2.34×10^{-90}	6. $\text{III} < \text{I} < \text{IV} < \text{II}$	2
7.	Consider the binary number $10\underline{1}100_{(2)}$. (a) Write down the place value of the underlined digit. (b) Express $101100_{(2)}$ in the expanded form. Answer for (b) : $1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0$	7. (a) 2^3	1 1
8.	Convert the decimal number $8^4 + 8^{11}$ into a hexadecimal number.	8. $200001000_{(16)}$	2
9.	If \$32 000 is deposited in a bank at a simple interest rate of 3% p.a., how many years will it take to receive an amount of \$34 880?	9. 3	2

10.	If the length and the width of a rectangle are increased by 10% and decreased by 30% respectively, find the percentage change in the area of the rectangle.	10. <u>-23%</u>	2										
11.	<p>The table below shows the salaries tax rates:</p> <table border="1" data-bbox="277 365 833 611"> <thead> <tr> <th>Net chargeable income</th> <th>Tax rate</th> </tr> </thead> <tbody> <tr> <td>On the first \$40 000</td> <td>2%</td> </tr> <tr> <td>On the next \$40 000</td> <td>7%</td> </tr> <tr> <td>On the next \$40 000</td> <td>12%</td> </tr> <tr> <td>Remainder</td> <td>17%</td> </tr> </tbody> </table> <p>(a) (i) If the net chargeable income is \$40 000, find the salaries tax payable. (ii) If the net chargeable income is \$80 000, find the salaries tax payable. (b) If David's salaries tax payable is \$4 200, find his net chargeable income.</p>	Net chargeable income	Tax rate	On the first \$40 000	2%	On the next \$40 000	7%	On the next \$40 000	12%	Remainder	17%	11. (a)(i) <u>\$800</u> (ii) <u>\$3 600</u> (b) <u>\$85 000</u>	1 1 2
Net chargeable income	Tax rate												
On the first \$40 000	2%												
On the next \$40 000	7%												
On the next \$40 000	12%												
Remainder	17%												
12.	It is given that $x < -\frac{1}{2}$ and $y = \frac{5}{6} - \frac{1}{3}x$. Find the range of the values of y .	12. <u>$y > 1$</u>	2										
13.	(a) Solve the inequality $\frac{2x+7}{3} > x+4$. (b) Represent the solution of (a) on the following number line. 	13. (a) <u>$x < -5$</u>	2 1										
14.	Determine whether each of the following statements must be true. Circle the correct answer. (a) If $a > b > c$, then $ab > bc$. (b) If $x > y > z$, then $x - y > y - z$. (c) If $p > q > r > 0$, then $\frac{p}{r} > \frac{q}{r}$.	14. (a) True <u>False</u> (b) True <u>False</u> (c) <u>True</u> / False	1 1 1										
15.	It is given a set of data: 15, 3, 10, 15, 22, 11, m , n . If the mode of the above set of data is 10, find the values of m and n .	15. $m =$ <u>10</u> $n =$ <u>10</u>	1 1										
16.	Consider the positive integers: 4, 4, 5, 6, x , y . If the mean of the above data is 4, which of the following is/are true? I. $x + y = 5$ II. Median = 4 III. Mode = 4	16. <u>I, II, III</u>	2										

Section B (60%)

All working must be clearly shown in the spaces provided.

17. Simplify $\left(\frac{-x^6y^{-3}}{x^{-2}y}\right)^3$ and express the answer with positive indices. (3 marks)

	$\left(\frac{-x^6y^{-3}}{x^{-2}y}\right)^3$	
	$= \frac{-x^{18}y^{-9}}{x^{-6}y^3}$	
	$= \frac{-x^{18-(-6)}}{y^{3+9}}$	
	$= \frac{-x^{24}}{y^{12}}$	

18. (a) Solve the inequality $\frac{2x+1}{5} - 1 < x + 2$. (3 marks)
- (b) Write down all negative integers satisfying the inequality in (a). (1 mark)

	<p>(a)</p> $\frac{2x+1}{5} - 1 < x + 2$ $2x+1-5 < 5x+10$ $-3x < 14$ $x > -\frac{14}{3}$	
	<p>(b) -1, -2, -3, -4</p>	

19. A shop produces cheese cakes and chocolate cakes only. The costs of producing one cheese cake and one chocolate cake are \$65 and \$35 respectively. If 140 cakes are produced on one day and the total cost should not exceed \$8000, at least how many chocolate cakes should be produced?

(4 marks)

Suppose x chocolate cakes should be produced.

$$65(140 - x) + 35(x) \leq 8000$$

$$9100 - 65x + 35x \leq 8000$$

$$-30x \leq -1100$$

$$x \geq \frac{110}{3}$$

At least 37 chocolate cakes should be produced.

20. In a cylinder, its base radius is 5 cm and volume is $150\pi \text{ cm}^3$. Find the total surface area of the cylinder. Give your answer in terms of π .

(4 marks)

Let the height of the cylinder = h cm

$$\pi(5)^2 h = 150\pi$$

$$h = 6$$

Total surface area

$$= 2 \times \pi(5)^2 + 2\pi(5)(6)$$

$$= 110\pi \text{ cm}^2$$

21. The value of a pair of earphones was \$800 in 2015 and its value has decreased at a fixed rate each year. In 2017, the value of the earphones decreased to \$648.

- (a) Find the decay factor of the value of the earphones. (2 marks)
- (b) Suppose the decay factor of the value of the earphones remains unchanged, find the value of the earphones in 2011. Give your answer correct to the nearest dollar. (2 marks)

(a) Let the decay factor be x ,
 $800x^2 = 648$
 $x = 0.9$
 \therefore The decay factor is 0.9.

(b) The value of the watch in 2011
 $= \$800 \div (0.9)^4$
 $= \$1219$, cor. to the nearest dollar

22. A carbon dioxide molecule consists of one carbon atom and two oxygen atoms. The weights of a carbon atom and an oxygen atom are 1.99×10^{-26} kg and 2.67×10^{-26} kg respectively.

(Express the answers of (a) and (b)(i) in scientific notation.)

- (a) Find the weight of a carbon dioxide molecule. (2 marks)
- (b) A carbon dioxide extinguisher contains 1.9791 kg of carbon dioxide.
 (i) Find the number of carbon dioxide molecules in the extinguisher.
 (ii) Find the total weight of the oxygen atoms in the extinguisher.

(4 marks)

(a) $1.99 \times 10^{-26} + 2(2.67 \times 10^{-26})$
 $= (1.99 + 2 \times 2.67) \times 10^{-26}$
 $= 7.33 \times 10^{-26}$ kg

(b)(i) $\frac{1.9791}{7.33 \times 10^{-26}}$
 $= 0.27 \times 10^{26}$
 $= 2.7 \times 10^{25}$

(ii) 2.67×10^{-26} kg $\times 2 \times 2.7 \times 10^{25}$
 $= 1.4418$ kg

23. The table below shows the marks that Mak and Millie got in various subjects in an examination and the weight of each subject.

	Chinese	English	Mathematics	P.E.
Mak	82	y	95	63
Millie	80	90	64	88
Weight	4	4	x	1

It is given that the weighted mean mark of Millie is 80.

- (a) Find x . (2 marks)
- (b) Given that the weighted mean mark of Mak is higher than that of Millie by 5 marks, find y . (3 marks)

(a)
$$\frac{80 \times 4 + 90 \times 4 + 64 \times x + 88 \times 1}{4 + 4 + x + 1} = 80$$

$$768 + 64x = 720 + 80x$$

$$16x = 48$$

$$x = 3$$

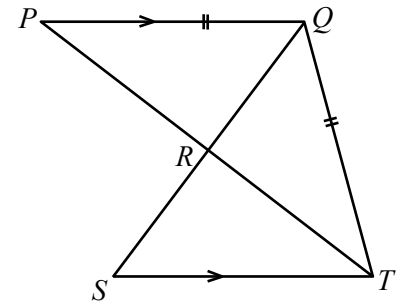
(b)
$$\frac{82 \times 4 + y \times 4 + 95 \times 3 + 63 \times 1}{4 + 4 + 3 + 1} = 85$$

$$676 + 4y = 1020$$

$$4y = 344$$

$$y = 86$$

24. In the figure, PRT and QRS are straight lines.
 $PQ = QT$ and $PQ \parallel ST$.



- (a) Prove that $\triangle PQR \sim \triangle TSR$. (3 marks)
 (b) Prove that PT is the angle bisector of $\angle QTS$. (3 marks)
 (c) If $PQ = ST$, is QR a perpendicular bisector of $\triangle PQT$? Explain your answer. (4 marks)

(a) In $\triangle PQR$ and $\triangle TSR$,
 $PQ \parallel ST$ (given)
 $\angle QPR = \angle RTS$ (alt. \angle s, $PQ \parallel ST$) any
 $\angle PQR = \angle TSR$ (alt. \angle s, $PQ \parallel ST$) two
 $\angle PRQ = \angle TRS$ (vert. opp. \angle s)
 $\therefore \triangle PQR \sim \triangle TSR$ (AA)

(b) $\angle QPT = \angle PTS$ (alt. \angle s, $PQ \parallel ST$)
 $PQ = QT$ (given)
 $\angle QTP = \angle QPT$ (base \angle s, isos. Δ)
 $= \angle PTS$ (proved)
 $\therefore PT$ is the angle bisector of $\angle QTS$.

(c) $ST = PQ$ (given)
 $= QT$ (given)
 $\angle QTR = \angle RTS$ (proved)
 $\therefore TR \perp QS$ (properties of isos. Δ)
 $PQ = QT$ (given)
 $QS \perp PT$ (proved)
 $\therefore PR = RT$ (properties of isos. Δ)
 $\therefore QR$ is a perpendicular bisector of $\triangle PQT$.

Alternative Method

$ST = PQ$ (given)
 $= QT$ (given)
 $\angle TQS = \angle TSQ$ (base \angle s, isos. Δ)
 $= \angle PQS$ (alt. \angle s, $PQ \parallel ST$)
 $PQ = QT$ (given)
 $\therefore QR \perp PT$ (prop. of isos. Δ)
 $\therefore PR = RT$ (prop. of isos. Δ)
 $\therefore QR$ is a perpendicular bisector of $\triangle PQT$.

25. Factorize

(a) $5a^2 - 8ab - 4b^2$, (1 mark)

(b) $25a^2 - 4b^2$, (1 mark)

(c) $100a^2x - 5a^2 + 4b^2 - 16b^2x + 8ab$. (4 marks)

(a) $5a^2 - 8ab - 4b^2 = (5a + 2b)(a - 2b)$

(b) $25a^2 - 4b^2 = (5a + 2b)(5a - 2b)$

(c) $100a^2x - 5a^2 + 4b^2 - 16b^2x + 8ab$
 $= 100a^2x - 16b^2x - 5a^2 + 8ab + 4b^2$
 $= 100a^2x - 16b^2x - (5a + 2b)(a - 2b)$
 $= 4x(25a^2 - 4b^2) - (5a + 2b)(a - 2b)$
 $= 4x(5a + 2b)(5a - 2b) - (5a + 2b)(a - 2b)$
 $= (5a + 2b)[4x(5a - 2b) - (a - 2b)]$
 $= (5a + 2b)(20ax - 8bx - a + 2b)$

26. In Figure 1, a piece of wire is bent into a sector of radius 12 cm and the angle of sector is 60° .

- (a) Find the area enclosed by the wire in Figure 1. (2 marks)
- (b) In Figure 2, the sector is reshaped into a circle. Someone claims that the area of the circle is smaller than the area of the sector. Do you agree? Explain your answer. (4 marks)

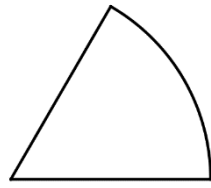


Figure 1

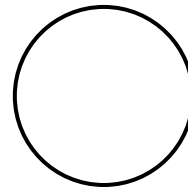


Figure 2

(a) The area enclosed by the wire

$$= \pi(12)^2 \times \frac{60^\circ}{360^\circ}$$

$$= \underline{24\pi \text{ cm}^2} \quad (\text{or } 75.4 \text{ cm}^2, \text{ corr. to 3 sig. fig.})$$

(b) The length of the wire

$$= 12 \times 2 + 2\pi(12) \times \frac{60^\circ}{360^\circ}$$

$$= (24 + 4\pi) \text{ cm}$$

The area of the circle

$$= \left(\frac{24 + 4\pi}{2\pi} \right)^2 \pi$$

$$\approx 106.4 \text{ cm}^2$$

$$> 24\pi \text{ cm}^2$$

\therefore I do not agree with the claim.

27. John deposits a fixed amount \$ x into an account at the beginning of each month in Bank A at an interest rate of 12% p.a. compounded monthly.

- (a) Find the total amount in the account at the end of the first month in terms of x . (2 marks)
- (b) It is given that the total amount in the account at the end of the third month is \$306 040.1.
- (i) Find x . (3 marks)
- (ii) Someone claims that the total amount in the account at the end of the **second** month can be exactly twice that of the amount in the account at the end of the **fourth** month if John deposits \$**2** x at the beginning of each month. Is the claim correct? Explain your answer. (3 marks)

(a) The total amount in the account at the end of the first month

$$= \$x \times \left(1 + \frac{0.12}{12}\right)$$

$$= \$1.01x$$

(b)(i) $[(1.01x + x) \times 1.01 + x] \times 1.01 = 306040.1$

$$(1.01^3 + 1.01^2 + 1.01)x = 306040.1$$

$$3.060401x = 306040.1$$

$$x = 100000$$

(b)(ii)

The amount in the account at the end of the fourth month

$$= \$((1.01^3 + 1.01^2 + 1.01)2x + 2x) \times 1.01$$

$$= \$ (1.01^4 + 1.01^3 + 1.01^2 + 1.01)2x$$

$$= \$ (1.01^4 + 1.01^3 + 1.01^2 + 1.01) \times 2(100000)$$

$$= \$ 820201.002$$

The amount in the account at the end of the second month

$$= \$ (1.01(2x) + 2x) \times 1.01$$

$$= \$ (1.01^2 + 1.01)2x$$

$$= \$ (1.01^2 + 1.01)2 \times (100000)$$

$$= \$ 406020$$

$$< \$ 410100.501 \quad (= \$ 820201.002/2)$$

\therefore The claim is not correct.

