St. Paul's College
F.2 Final Examination 2021-2022

MATHEMATICS
PAPER 1

Section A Question-Answer Book

Time Allowed: 1 hour 15 minutes

A

Name: ________________________ Class: ______ Class Number: ____ Marks: _____________

INSTRUCTIONS

1. Answer ALL questions.
2. Answer this section in the spaces provided in this Question-Answer Book.
3. All working must be clearly shown.
4. Marks will be deducted for poor and untidy presentation.
5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
6. The diagrams in this paper are not necessarily drawn to scale.

Section A (40 marks)

1. Simplify $\frac{(2x^4y^3)^2}{x^2y^4}$. (3 marks)

2. (a) Simplify $\sqrt{175}$. (1 mark)

(b) Rationalize the denominator of $\dfrac{21}{4\sqrt{7}}$. (2 marks)

(c) Hence, simplify $\dfrac{21}{4\sqrt{7}} - \dfrac{\sqrt{175}}{20} + \dfrac{\sqrt{7}}{5}$. (2 marks)

3. The volume of a bottle of milk is measured as 540 mL correct to the nearest 10 mL.

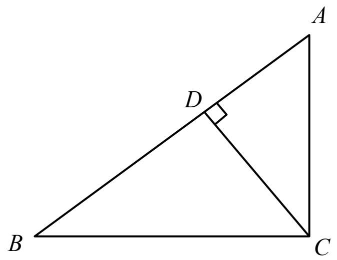
(a) Find the maximum absolute error of the measured volume. (2 marks)

(b) Find the percentage error of the measured volume. (2 marks)

4. (a) Factorize $16x^2 - y^2$. (1 mark)

(b) Using the result from (a), simplify $1 - \dfrac{16x^2 - y^2}{2x(4x + y)}$. (3 marks)

5. In the figure, D is a point lying on AB such that $CD \perp AB$. It is given that $\angle BAC = 52°$, $AD = 6$ cm and $BC = 12$ cm.



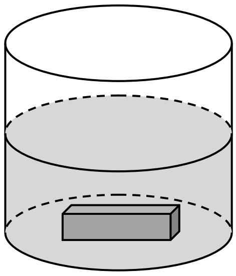
(a) Find CD correct to 3 significant figures. (2 marks)

(b) Is $\triangle ABC$ an acute-angled triangle? Explain your answer. (3 marks)

6. In the figure, a right circular cylindrical tank of base radius 12 cm contains $1080\pi\ \text{cm}^3$ of water.

(a) Find the depth of water in the tank. (2 marks)

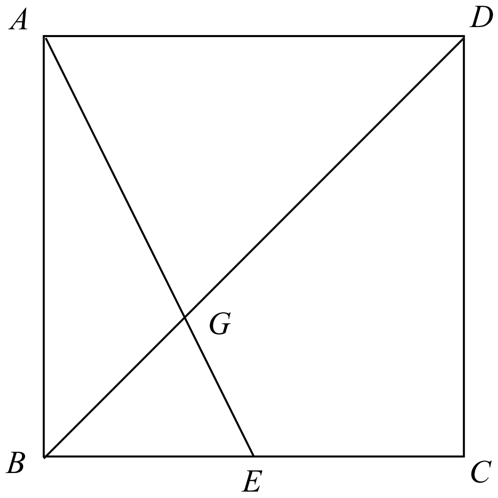
(b) When a block is put in the tank and it is totally immersed in water, the rise in water level is 1.5 cm, find the volume of the block in terms of π. (2 marks)



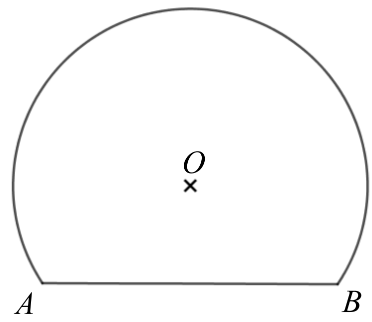
7. It is given that $ABCD$ is a square. E is the mid-point of BC and G is the intersection of AE and BD.

(a) Prove that $\triangle AGD \sim \triangle EGB$. (3 marks)

(b) If $AB = 6$ cm, using the result in (a), find AG. (4 marks)



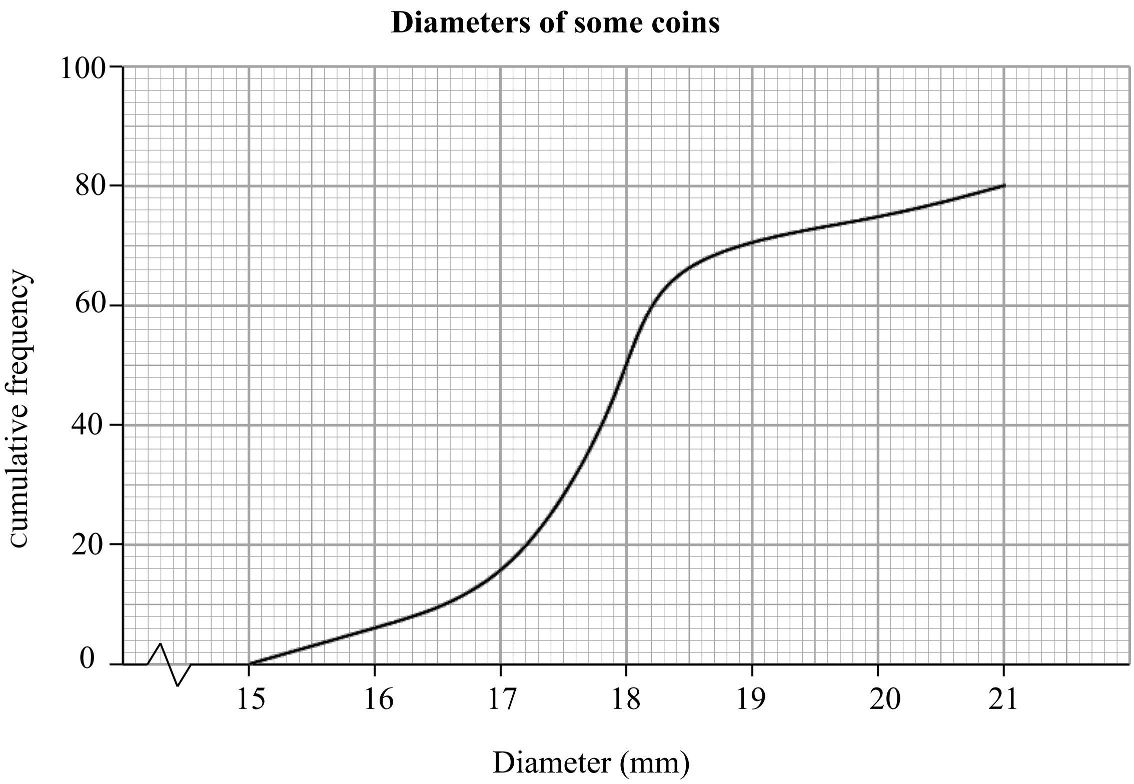
8. A straight tunnel has a uniform cross section as shown in the figure below. The cross section is in the form of a segment with centre O and radius $3\,\text{m}$. Let A and B be the end points of the chord representing the road. It is given that $\angle AOB = 160°$.



(a) Find the length of the arc AB of the cross section. (2 marks)

(b) If the curved surface of the tunnel is painted and the cost of paint is $\$4/\text{m}^2$, find the total cost of the paint for painting a tunnel $2\,\text{km}$ long. (2 marks)

9. The following cumulative frequency curve shows the distribution of the diameters of some coins.



(a) Write down the median, the lower quartile and the upper quartile of the distribution.

(3 marks)

(b) How many coins have diameters less than 17 mm? (1 mark)

End of Section A

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B

MATHEMATICS
PAPER 1

Section B Question-Answer Book

Time Allowed: 1 hour 15 minutes

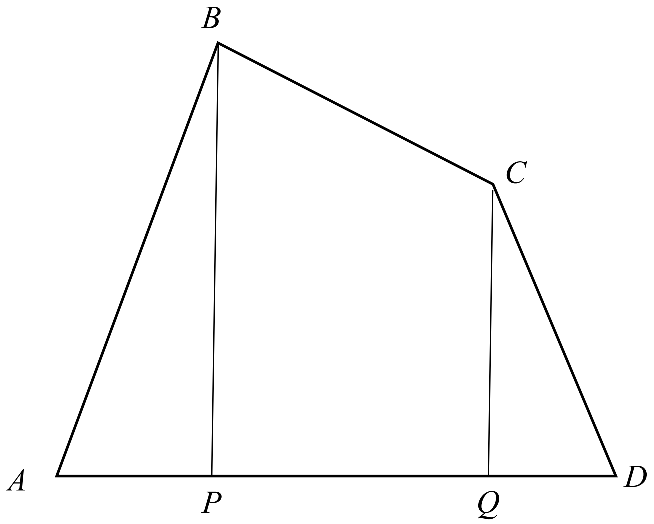
Name: ________________________ Class: ______ Class Number: ____ Marks: _____________

INSTRUCTIONS

1. Answer ALL questions.
2. Answer this section in the spaces provided in this Question-Answer Book.
3. All working must be clearly shown.
4. Marks will be deducted for poor and untidy presentation.
5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
6. The diagrams in this paper are not necessarily drawn to scale.

Section B (30 marks)

1. The figure shows a quadrilateral $ABCD$. $AB = 8$ cm, $BC = 5$ cm and $AD = 9$ cm. $\angle DAB = 70^\circ$, $\angle ABC = 85^\circ$. P and Q are points on AD such that BP and CQ are both perpendicular to AD. Find, correct to 3 significant figures,



(a) AP and BP (2 marks)

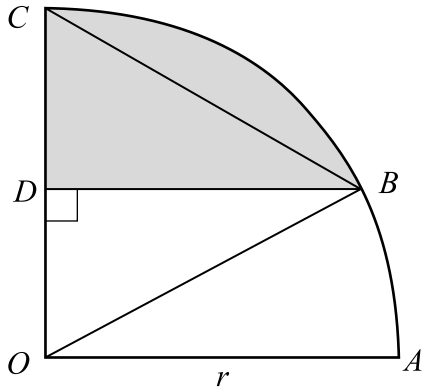
(b) PQ (2 marks)

(c) CQ (2 marks)

(d) CD (2 marks)

(e) $\angle CDA$ (2 marks)

2. The figure shows a sector *OABC* with centre *O*, radius *r* and $\angle COA = 90^\circ$. *D* is the mid-point of *OC* and *BD* is perpendicular to *OC*.



(a) Show that $\angle COB = 60^\circ$. (2 marks)

(b) Show that $BD = \frac{\sqrt{3}}{2}r$. (2 marks)

(c) Find the area of the shaded region *BCD*, express your answer in terms of *r* and π. (4 marks)

3. (a) Factorize $(m^2 - n^2)^2 + (2mn)^2$. (3 marks)

(b) It is given that θ is acute and $\sin\theta = \dfrac{2mn}{m^2 + n^2}$, where m and n are positive numbers and $m > n$. Using (a) or otherwise, find $\cos\theta$ in terms of m and n. (3 marks)

(c) If $m^2 + n^2 = 41$ and $mn = 20$, where m and n are positive numbers and $m > n$.

(i) Find the values of $(m + n)^2$ and $(m - n)^2$.

(ii) Using (a), (c)(i) or otherwise, find m and n. (6 marks)

End of Section B

End of Paper

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MATHEMATICS
PAPER 1

Section A Solution

Time Allowed: 1 hour 15 minutes

A

Max pp-1 or u-1 for this section

	Solutions	Marks	Remarks
1.	$\dfrac{(2x^4y^3)^2}{x^2y^4}$ $=\dfrac{4x^8y^6}{x^2y^4}$ $=4x^6y^2$	2M 1A	1M for $(ab)^m=a^mb^m$ or $(a^m)^n=a^{mn}$ 1M for $a^m\times a^n$ or $\dfrac{a^m}{a^n}=a^{m-n}$
2a.	$\sqrt{175}$ $=\sqrt{25\times 7}$ $=5\sqrt{7}$	1A	
2b.	$\dfrac{21}{4\sqrt{7}}$ $=\dfrac{21}{4\sqrt{7}}\times\dfrac{\sqrt{7}}{\sqrt{7}}$ $=\dfrac{21\sqrt{7}}{28}$ $=\dfrac{3\sqrt{7}}{4}$	1M 1A	
2c.	$\dfrac{21}{4\sqrt{7}}-\dfrac{\sqrt{175}}{20}+\dfrac{\sqrt{7}}{5}$ $=\dfrac{3\sqrt{7}}{4}-\dfrac{5\sqrt{7}}{20}+\dfrac{\sqrt{7}}{5}$ $=\dfrac{15\sqrt{7}-5\sqrt{7}+4\sqrt{7}}{20}$ $=\dfrac{14\sqrt{7}}{20}$ $=\dfrac{7\sqrt{7}}{10}$	1M 1A	

3a.	Maximum abs error: $10\times\frac{1}{2}$ $=5mL$	1M 1A	Can be absorbed
3b.	Percentage error: $\frac{5}{540}\times 100\%$ $=0.926\%(3sf)$	1M 1A	$\approx 0.926\%$
4a.	$16x^2-y^2$ $=(4x-y)(4x+y)$	1A	
4b.	$1-\frac{16x^2-y^2}{2x(4x+y)}$ $=1-\frac{(4x+y)(4x-y)}{2x(4x+y)}$ $=1-\frac{4x-y}{2x}$ $=\frac{2x-4x+y}{2x}$ $=\frac{-2x+y}{2x}$ Or $=1-\frac{(4x+y)(4x-y)}{2x(4x+y)}$ $=1-\frac{4x-y}{2x}$ $=1-2+\frac{y}{2x}$ $=-1+\frac{y}{2x}$	1M 1M 1A o.e. 1M 1M 1A	For applying (a)
5a.	In ΔACD, $\tan 52^\circ=\frac{CD}{6}$ $CD=6\tan 52^\circ$ $CD=7.68cm(3sf)$	1M 1A	Either $CD\approx 7.68cm$
5b.	$\sin\angle CBD=\frac{6\tan 52^\circ}{12}$ or $\cos\angle BCD=\frac{6\tan 52^\circ}{12}$ $\angle CBD=39.8^\circ(3sf)$ $\angle BCD=50.2^\circ(3sf)$ In ΔABC, $\angle ABC+\angle BCA+\angle CAB=180^\circ$ ($\angle$ sum of $\triangle$) $\angle BCA=88.2^\circ(3sf)$ $<90^\circ$ $\therefore$ The claim is agreed.	1M 1A 1A f.t.	Must compare

6a.	Let h cm be the original depth of the water in the tank, $12^2 \times \pi \times h = 1080\pi$ $h = 7.5$ $\therefore$ The original depth of water in the tank is 7.5cm.	1M 1A	
6b.	The volume of the block, $12^2 \times \pi \times 1.5$ $= 216\pi cm^3$	1M 1A	
7a.	In ΔAGD and ΔEGB, $\angle GAD = \angle GEB$ (alt. $\angle$s, AD//BC) $\angle ADG = \angle EBG$ (alt. $\angle$s, AD//BC) $\angle AGD = \angle EGB$ (vert. opp. $\angle$s) $\therefore \Delta AGD \sim \Delta EGB$ (A.A.A.) Marking scheme: Case 1: Any correct proof with correct reasons. Case 2: Any correct proof without reasons. Case 3: Any correct argument with correct reason.	3 2 1	
7b.	$\because AB = AD = 6$ $\Delta AGD \sim \Delta EGB$ $\frac{AD}{EB} = \frac{AG}{EG}$ (corr. sides, ~ Δs) $\frac{AG}{EG} = \frac{2}{1}$ $\therefore$ AG : EG = 2 : 1 In ΔABE, $AE^2 = 6^2 + 3^2$ (Pyth. Thm.) $AE = 3\sqrt{5}$ $\therefore AG = \frac{2}{3} \times 3\sqrt{5}$ $= 2\sqrt{5} cm$	1M 1M 1M 1A	Must incl. both reasons
8a.	Length of arc AB, $\frac{360° - 160°}{360°} \times 2 \times 3\pi$ $= \frac{10\pi}{3}$ $= 10.5m(3sf)$	1M 1A	Either
8b.	The cost of the paint for painting a 2 km tunnel, $\frac{10\pi}{3} \times 2 \times 1000 \times 4$ $= \$83800(3sf)$	1M 1A	
9a	Lower Quartile: 17.2 mm Median: 17.8 mm Upper Quartile: 18.2 mm	1A 1A 1A	
9b.	From the graph, the number of coins that have a diameter less than 17mm is 16.	1A	

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MATHEMATICS
PAPER 1

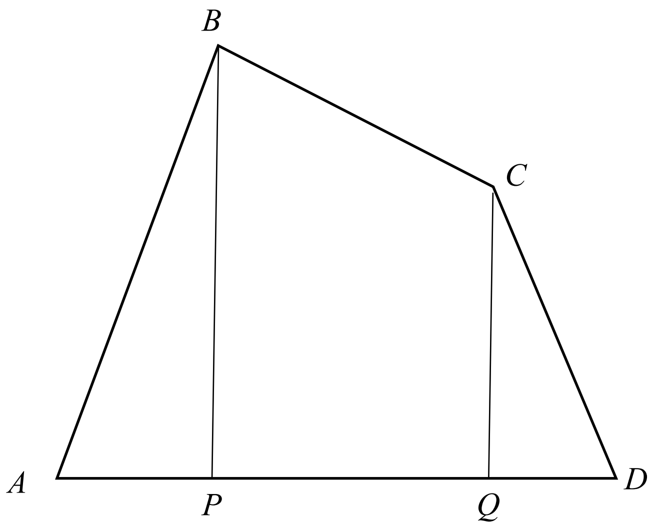
B

Section B Marking Scheme

Time Allowed: 1 hour 15 minutes

Section B (30 marks)

1. The figure shows a quadrilateral $ABCD$. $AB = 8$ cm, $BC = 5$ cm and $AD = 9$ cm. $\angle DAB = 70^\circ$, $\angle ABC = 85^\circ$. P and Q are points on AD such that BP and CQ are both perpendicular to AD. Find, correct to 3 significant figures,



(a) AP and BP (2 marks)
(b) PQ (2 marks)
(c) CQ (2 marks)
(d) CD (2 marks)
(e) $\angle CDA$ (2 marks)

(a) $AP = AB \cos \angle BAP = 8 \cos 70^\circ = 2.74$ cm (3 s.f.)	**1A**
$BP = AB \sin \angle BAP = 8 \sin 70^\circ = 7.52$ cm (3 s.f.)	**1A**
(b) Let X be the foot of perpendicular from C to BP. (X on BP, $CX \perp BP$)	
$\angle CBX = \angle ABC - \angle ABP = 85^\circ - (90^\circ - 70^\circ) = 65^\circ$	**1**
$PQ = XC = BC \sin \angle CBX = 5 \sin 65^\circ = 4.53$ cm (3 s.f.)	**1A**

(c) $CQ = XP = BP - BX = 8 \sin 70^\circ - 5 \cos 65^\circ = 5.40$ cm (3 s.f.) **1+1A**

(d) $QD = AD - AP - PQ = 9 - 8 \cos 70^\circ - 5 \sin 65^\circ = 1.73$ cm **1A**

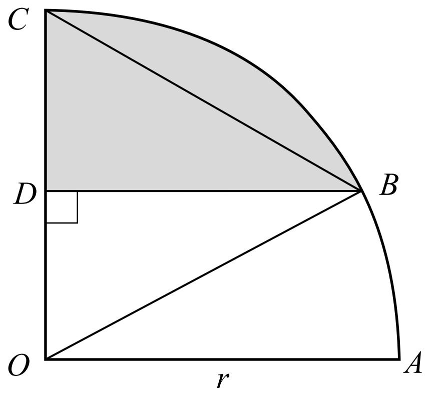
$CD^2 = CQ^2 + QD^2 = (8 \sin 70^\circ - 5 \cos 65^\circ)^2 + (9 - 8 \cos 70^\circ - 5 \sin 65^\circ)^2$

so $CD = 5.68$ cm (3 s.f.) **1A**

(e) $\tan\angle CDA = \dfrac{CQ}{QD}$ **1M**

So $\angle CDA = 72.2^\circ$ (3 s.f.) **1A**

2. The figure shows a sector $OABC$ with centre O, radius r and $\angle COA = 90^\circ$. D is the mid-point of OC and BD is perpendicular to OC.



(a) Show that $\angle COB = 60^\circ$. (3 marks)

(b) Show that $BD = \dfrac{\sqrt{3}}{2} r$. (2 marks)

(c) Find the area of the shaded region BCD, express your answer in terms of r and π. (4 marks)

(a) $CD = DO$ (given)

$\angle CDB = \angle ODB = 90^\circ$ (given),

$DB = DB$ (common)

So, $\Delta CDB \cong \Delta ODB$ (SAS) **1 (SAS or Pyth. Thm)**

So, $CB = OB$ (corr. sides, $\cong \Delta$s) **1**

As, $OC = OB = r$, ΔCBO is an equilateral triangle

So $\angle COB = 60^\circ$ (property of equilateral Δ) **1**

(b) $BD^2 + OD^2 = BO^2$ (Pyth. Thm) **1M**

$BD^2 + (\frac{r}{2})^2 = r^2$,

So $BD^2 = \frac{3r^2}{4}$ and so $BD = \frac{\sqrt{3}r}{2}$ cm **1 f.t.**

(c) the area of the shaded region BCD

= area of sector OCB – area of triangle ODB **1**

$= \pi r^2 \times \frac{60°}{360°} - \frac{1}{2} \cdot \frac{r}{2} \cdot \frac{\sqrt{3}r}{2}$ **1A + 1A**

$= \frac{\pi}{6} r^2 - \frac{\sqrt{3}}{8} r^2$ **1A**

$= \frac{4\pi - 3\sqrt{3}}{24} r^2$

3. (a) Factorize $(m^2 - n^2)^2 + (2mn)^2$. (2 marks)

(b) It is given that θ is acute and $\sin\theta = \frac{2mn}{m^2 + n^2}$, where m and n are positive numbers and $m > n$. Using (a) or otherwise, find $\cos\theta$ in terms of m and n. (3 marks)

(c) If $m^2 + n^2 = 41$ and $mn = 20$, where m and n are positive numbers and $m > n$.

(i) Find the values of $(m + n)^2$ and $(m - n)^2$.

(ii) Using (a), (c)(i) or otherwise, find m and n. (6 marks)

(a) $(m^2 - n^2)^2 + (2mn)^2$

$= m^4 - 2m^2n^2 + n^4 + 4m^2n^2$ **1**

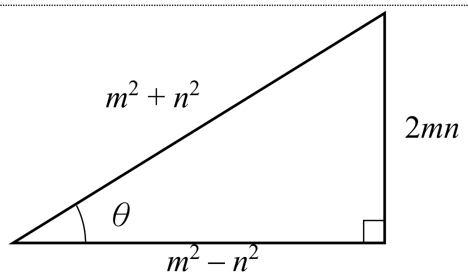
$= m^4 + 2m^2n^2 + n^4$

$= (m^2 + n^2)^2$ **1A**

(b) From (a), $(m^2 - n^2)^2 + (2mn)^2 = (m^2 + n^2)^2$ **1M**

a right-angle triangle with the opposite side is $2mn$ and the hypotenuse is $m^2 + n^2$

then the adjacent side is $m^2 - n^2$, $(m > n)$ **1**



Hence if $\sin\theta = \frac{2mn}{m^2 + n^2}$, then $\cos\theta = \frac{m^2 - n^2}{m^2 + n^2}$ **1A**

(c) (i) $(m + n)^2 = m^2 + 2mn + n^2$ **1M**

$= (m^2 + n^2) + 2mn = 41 + 2(20) = 81$ **1A**

$(m-n)^2 = m^2 - 2mn + n^2 = (m^2 + n^2) - 2mn = 41 - 2(20) = 1$ **1A**

(ii) from (i), $(m+n)^2 = 81$,

so $m + n = 9$ or $m + n = -9$

But since both m and n are positive, $m + n$ is also positive.

So $m + n = 9$

From (i), $(m-n)^2 = 1$

so $m - n = 1$ or $m - n = -1$ either **1**

But since $m > n$, so $m - n$ is positive.

So $m - n = 1$

Solving $\begin{cases} m+n=9 \\ m-n=1 \end{cases}$, **1**

we have $m = 5$, $n = 4$ **1A**

[alternatively, using (a) we have $(m^2 - n^2)^2 = (m^2 + n^2)^2 - 4m^2n^2 = (41)^2 - (40)^2 = 81$

So $m^2 - n^2 = 9$ or -9

but both m and n are positive and $m > n$, so $m^2 > n^2$.

Hence $m^2 - n^2 = 9$ **1**

Solving $\begin{cases} m^2+n^2=41 \\ m^2-n^2=9 \end{cases}$ **1**

we have $m^2 = 25$ and $n^2 = 16$

Since m and n are positive, $m = 5$, and $n = 4$ **1A]**

End of Section B

End of Paper